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A good picture of this arrangement is shown in [15]. Therefore $$a \sin (\theta - \delta_{o}) \; + \; s \cos \sigma \; + \; a\sin (\theta + \delta_{i}) \quad = \quad w$$ If we differentiate both sides of equation 40 with respect to δ_{o} we get $$a \sin (\theta - \delta_{o}) \; - \; s \sigma' \cos \sigma \; + \; a \delta_{i}' \sin (\theta + \delta_{i}) \quad = \quad 0$$ Similarly, differentiating equation 41, we get $$- a\cos (\theta - \delta_{o}) \; - \; s \sigma' \sin \sigma \; + \; a \delta_{i}' \cos (\theta + \delta_{i}) \quad = \quad 0$$ We can eliminate σ' from equation 42 and equation 43 and after some rearrangement we get $$\sin \sigma \left[\sin (\theta - \delta_{o}) \, + \, \delta_{i}' sin (\theta + \delta_{i}) \right] \; - \; \cos \sigma \left[- \cos (\theta - \delta_{o}) \, + \, \delta_{i}' \cos (\theta + \delta_{i}) \right] \;\; = \;\; 0$$ To evaluate δ_{i}' in the central steering position we put $\sigma = \delta_{o} = \delta_{i} = 0$ in equation 44, which after some rearrangement leads to $$\delta_{i}'(0) \quad = \quad 1$$ Now we differentiate equation 44 again. The friction drag is roughly proportional to the total surface area, and the shape of the vehicle does not particularly matter. You can think of it as surface friction. As shown in figure 16, which is based on data tabulated some years ago by R H Barnard [6], pressure drag is usually the largest single component. By injecting an input at the natural frequency you have excited resonance, and the frequency should be at the natural frequency of the system, i.e., the value noted during the preliminary test. The solution is to rake the pivot axis astern like the mast of a sailing ship, at an angle of several degrees from the vertical so it passes down through the wheel bearing to meet the road surface a short distance ahead of the centre of contact of the tyre with the road (figure 5). This vision of 20thcentury travel was made possible by soft coil springs together with wheels that were small and light compared with the vehicle they carried; the wheels bounced up and down over the bumps while the body sailed on. The semi-zeppelin However this doesn't take into account the equipment such as wheels and suspension needed to make the vehicle roadworthy. This is because the greater static deflection reduces the natural frequency of the suspension springs (see equation 2 in Section G1115). This has the advantage that the inside wheel then swivels through a smaller angle at full lock, which in turn reduces size of the wheel housing where it intrudes into the passenger cabin. It can be replaced by an equivalent d'Alembert force of value MV^{2}/R acting laterally towards the outside of the curve, and this is what we normally describe as 'centrifugal force'. The resultant now exerts a relatively small torque about the centre of mass, and the car is more likely to remain on its original path (figure 14). The resulting curve shows that the body returns to its equilibrium position quite quickly. In the 1840s, many steam locos capable of 60 mph had no brakes [11]. Also the wheels tended to foul the undercarriage (figure 2), and a jarring bump in the road would knock the steering out of line. Unfortunately, its hooves are in the wrong place. But we can put them into a similar form by converting each into a power series expansion that is asymptotically true for small δ_{o}. In fact, the designer really wants negative lift, and this is the reason why racing cars are fitted with inverted wings that press them down onto the road (see Section C1717). One might want the vehicle to turn into the wind like a weathercock so that it steers itself automatically, as it were, back towards its original path. It is this residual output from the spring and damper that makes travel less comfortable than it might be, and for many years, engineers have considered the possibility of an intelligent system whose output never varies. Using equation 55 we can also substitute for $h''(0)$ in equation 64, at the same time putting $\delta_{i}'(0) = 1$, and after some rearrangement we get $$\delta_{i}''(0) \quad = \quad 2 \tan (\theta - \phi) \left[1 \; + \; \frac{a}{b} \sin^{2} \theta \sec^{2} \phi \, \text{cosec} (\theta - \phi) \right]$$ Revised 19 February 2015 Page 8Have you ever watched horses perform at a circus? If you suspend a mass M from an idealised spring the mass will undergo 'simple harmonic motion', oscillating up and down at a certain characteristic frequency, rather like a pendulum. Likewise when a cyclist brakes, the horizontal forces acting on the bicycle wheels are directed rearwards not forwards, and there is nothing to cancel them out. Hysteresis in the tyre walls causes the centre of normal pressure to move forward of the axle centreline (see Section G1619). The effect, as with any form of oversteer (see Section C0418), can be very disconcerting for the driver. If a car were subjected to bumps at regular intervals corresponding to this frequency, the passengers would be very uncomfortable indeed, and in fact they would be better off with no springs at all. If you want to move a horse sideways (or in any direction, for that matter) the best place to push is higher up, at its centre of mass. For braking mechanisms that belong to our second category, 'rheostatic' brakes, there is no frictional contact at all. The last condition occurs when the car travels round a curve, forcing the outer wheel into jounce and the inner wheel into rebound. Turning to the rear of the vehicle we observe the third and most dramatic feature for a modern saloon: the two trailing vortices formed on either side of the rear pillars together with a turbulent wake of across the whole rear of the car (figure 19). Suspension kinematics The purpose of a car suspension is to allow each wheel move freely up and down over bumps and potholes in the road. But to a first approximation, suspension systems on cars, trains and aircraft only move up and down, and in this sense, they only do half the job. Nevertheless, inertia forces often appear in textbooks, where they are used to explain the dynamics of moving vehicles. An ideal aerodynamic shape The torpedo-shaped speedster 'Jamais contente' It happens that a circular cross-section is not very convenient for a passenger vehicle because it doesn't have a flat floor, but the zeppelin outline can be used in a different way, as a body of constant cross-section that stands upright like a shark's fin, with the airflow dividing symmetrically along either side. The ideal car Revised 16 February 2015Page 9 We use cookies on this website to maintain your browsing session and to improve the ways you use it.You can choose what types of cookies you consent to on this site via your Cookie Settings.Otherwise, if you're happy to consent to all cookies we use you can accept and carry on.You can modify your cookie settings at any time via the Cookie Settings link at the bottoms of the page.More information about how we handle personal data can be found in our Privacy Policy. We know that a force must be applied to make an object accelerate or decelerate, and when translated into the language of fluid flow this implies pressure changes along the stream tube. We have an 'accelerating system'. Braking will not only slow the car down, but also exert a couple about the car's centre of mass. The value of c for a car suspension might be around 1500 Ns/m. Fixed parameters for the four-bar linkage For any given model of car, there is not much freedom to change the wheelbase or the track width once the design process has begun. The French mathematician and philosopher Jean-Baptiste le Rond d'Alembert (1717-1783) showed that an accelerating system could be transformed into an equivalent static system by adding a suitable inertia force. Here, the wheel is forced to move up and down by a specified amount, which alters the deflection of the spring and damper. Other things being equal, if you are buying a car you will want it to look to look fast and powerful, and while fashions change from year to year, manufacturers have always used styling to create an impression of speed. At first sight these requirements seem contradictory, but in fact they refer to two different things. As explained in section G1114, its value is determined mostly by the spring stiffness. The early coach had no springs, and the passenger compartment hung by leather straps from a vertical post at each corner of the under-carriage (figure 1). This behaviour parallels the effect of low-frequency waves in the road surface: the output is exactly the same as the input. In the case of a motor vehicle, this axis happens to coincide roughly with the kingpin axis, so the wheel tends to steer out of line. The steering became unpredictable, the front wheels tended to hop when the brakes were applied, and in some cases the front suspension would shimmy until either the driver slowed down or the car shook itself to pieces. Like many other sports cars of the day, the body was said to generate a certain amount of lift at the rear axle, and a fin was added to the prototype to improve directional stability. Rounding the A-pillars in horizontal cross-section reduces the intensity of the trailing vortices [15], although for a long, box-shaped vehicle, a comparatively small radius will do [35]. Most of the time, the front wheels of your family saloon point within a degree or two on either side of the straight ahead position. You have only to turn the handwheel a few degrees in either direction to make the car veer sharply across several lanes, and possibly turn over. We need (a) springs, and (b) dampers. The air, which is more-or-less incompressible at road traffic speeds, must flow faster as it passes through the constriction. At high speeds, the assumptions underlying the 'pure rolling' analysis no longer apply, because the wheels don't move in the direction they are pointing. It is still an important priority for light vehicles, and partly accounts for the introduction of alloy wheels on 'sporty' models. Making a system work along these lines is far from easy [16]. Vortices of this type are known as trailing vortices because they extend rearwards into the atmosphere. Separation can also arise round both sides of the windshield to form trailing vortices that extend rearward into the wake from the A-pillars as shown in figure 18. The values of $\delta_{ip}'(0)$ and $\delta_{ip}''(0)$ are derived in Appendix 1, equation 32 and equation 34. Should the external power fail, steering is lost and the driver has no control. And the severity of the impact increases with increasing speed. To manoeuvre in or out of a parking space, the driver must pull hard on the handwheel because when a front wheel swivels about its pivot axis, the contact patch is forced to rotate around its centre, scrubbing the road surface (figure 12). Double wishbone suspension Of all the modern varieties of suspension, the transverse double wishbone (figure 1, figure 16) is nowadays regarded as state-of-the-art not only for passenger cars but for buses as well. Ideally, therefore, to provide an equally smooth ride under all loading conditions, the stiffness of the springs should vary according to the number of passengers and the load carried. So if the road surface waves have an amplitude of 10 mm, the body vibration will have an amplitude of about 0.16 $\times$ 10 mm, or 1.6 mm. Trams, on the other hand, are not protected by a block signalling system, and because they share road space with private traffic, they experience sharp deceleration more frequently. But occasionally in a strong crosswind the angle can reach 20°. We can illustrate this using the quarter-vehicle model with suspension parameter values equal to the ones used in the previous example ($\zeta =$ 0.4 and $\omega =$ 6.28 rad/s, with a sinusoidal road surface input of amplitude 10 mm at a frequency of 5 Hz). The modifications can be broken down into three main areas: front end, rear end, and underbody plus engine compartment. They are light and compact, which is helpful particularly in the restricted space underneath the wheel arch of a small car (Section C1405). By rearranging equation 27 we see that the equation for the inner steer angle in pure rolling is $$\delta_{ip} \quad = \quad \text{arccot} \left(\cot \delta_{o} \; - \; \frac{w}{L} \right)$$ The actual relationship for the 4-bar link is derived in Appendix 2, equation 35 to equation 38, which in a slightly modified form can be written: $$\delta_{i} \quad = \quad \frac{\pi }{2} \; \; - \; \; \alpha \;\; - \;\; \beta \;\; - \;\; \theta$$ where $$\alpha \quad = \quad \arccos \, \left[\frac{e^{2} \; + \; w^{2} \; - \; a^{2}}{2ew} \right]$$ $$\beta \quad = \quad \arccos \, \left[{\frac{e^{2} \; + \; a^{2} \; - \; s^{2}}{2ea} } \right]$$ and $$e^{2} \quad = \quad a^{2} \; + \; w^{2} \; - \; 2aw \sin (\theta \, - \, \delta _o)$$ We now know the value of the inner steering angle required for the pure rolling condition, equation 6, and also the inner steering angle for the actual linkage, equation 7. Hence if the road surface input is a regular sine wave of amplitude a metres at a frequency of Ω rad/s, the quarter-body oscillates at the same frequency but with a different amplitude whose value depends on Ω. Parallel and reciprocal motions First, though, it's necessary to point out that the behaviour of vehicle suspension cannot be understood entirely in terms of the behaviour of a single wheel moving in isolation. What makes passengers comfortable Every car suspension has a natural frequency of vibration, the frequency at which the body will bounce up and down, oscillating like a giant tuning fork. Under emergency braking, the load distribution between front and rear axles on a typical car changes very roughly from 60:40 to 80:20 [1] [2]. Hold the loose end of the rubber band in one hand and let the teacup hang down well clear of the floor. Offset Now let us focus, say, on the left-hand front wheel, viewed from the front. If the wavelength is L, a vehicle moving at speed V will encounter a peak every L/V seconds, and the frequency is therefore V/L cycles per second (hertz). An important consideration is to restrict the air flow to the minimum necessary for cooling, which in turn implies a small inlet. 'Input' means up-and-down movements of the wheel as it is jolted by roughness and bumps in the running surface, whose profile can be pictured as a superposition of sine waves of many different frequencies. The boundary layer is of special interest because it behaves differently from the bulk of the air around the vehicle, being slowed down by friction. The reason was nothing to do with stability. Early cars suffered from 'shimmy': rapid oscillations around the kingpin axis as if the driver were tugging the handwheel alternately left and right in quick succession [2] [15]. Most cars experience a great deal of air resistance, far more than the theoretical minimum for an aerodynamically efficient body of the same overall size. Proc Conf on Light Rail '93, 23-25 November 1993, Birmingham (L Lesley, editor) Liverpool: Transport Science. Until recently, most power-assisted systems were hydraulic, with the extra steering torque provided via a hydraulic pump driven by the engine. Unlike tyres, which rely on the weight of the vehicle to supply the normal contact force, a brake pad is pressed against the disk under hydraulic pressure, which within reason can be increased to any desired level. A high rear end helps, although if the basic dimensions have already been fixed it may be quite difficult to make the vehicle stable through minor changes in the body shape; occasionally in the past, it was not unknown for a designer to add a fin to a sports racing car as a last resort. The larger the cross-sectional area of the wake, the greater the drag. The Maclaurin expansion for δ_{ip} is $$\delta_{ip} \quad = \quad \delta_{ip}(0) \;\; + \;\; \frac{\delta_{o}}{1!}\delta_{ip}'(0) \;\; + \;\; \frac{\delta_{o}^2}{2!} \delta_{ip}''(0) \;\; + \;\; \text{terms in powers of } \delta_{o}^3 \text{ and above}$$ The Maclaurin expansion for δ_i is $$\delta_{i} \quad = \quad \delta_{i}(0) \;\; + \;\; \frac{\delta_{o}}{1!} \delta_{i}'(0) \;\; + \;\; \frac{\delta_{o}^2}{2!} {\delta_{i}}''(0) \;\; + \;\; \text{terms in powers of } \delta_{o}^3 \text{ and above}$$ It only remains to work out values for the coefficients. The result can loosely be summed up as follows: $$\mathrm{mass} \; + \mathrm{energy} \; + \; \mathrm{springs} \quad = \quad \mathrm{vibration}$$ Automobile pioneers discovered this early on with the suspension systems they had adapted from horse-drawn carriages. And there is another complication. There is scope for adapting to the needs of different vehicles by changing the lengths and the angles of the wishbone arms. Since air resistance increases as the square of speed, they can be effective on vehicles travelling over 100 km/h. The current is fed to resistors on the train roof, where the energy is dissipated as heat, or in some cases, returned to the overhead electrical supply (a schematic diagram is shown in figure 3). Eccentric loading of the suspension Raked pivot axis The angle of rake has increased over time. The drag is the component that acts against the direction of motion, so good aerodynamic design can make any car go faster, or save fuel, or achieve some combination of the two. It turns out that complete 'isolation' is not achievable with a mechanical suspension system along the lines indicated here. The relative movement at each joint increases roughly in proportion to the size of the load, and as it does so, dissipates energy through friction. When a vehicle moves, the distribution of weight among the wheels doesn't change because gravity continues to act in the same way regardless of what the driver does. Many railway trains have complex and sophisticated braking systems, especially intercity passenger sets, some of which travel three times faster than a typical family saloon. Roadholding So much for the passengers. Clearly, you don't want to drive along a road with a wavy surface that injects a pulse into the suspension at the same rate as the natural frequency because the motion could build up to an intolerable level. In the case of the Ford Sierra, the problem was cured by added a raised plastic strip to the roof pillar, which provided a trigger and thereby anchored the separation to a fixed point on the body surface [20]. The two paths are different. The boundary layer also exerts a normal force on every element of the vehicle body surface Relationship between speed and pressure within a stream tube To summarise, we observe an array of shear and normal pressure forces acting on the vehicle body surface. This is why dampers for cars are usually made with response characteristics that vary on different parts of the suspension stroke, with the value of the damping coefficient on rebound about three times the value on jounce [2] [4]. Energy dissipated by different types of vehicle while braking Category Distance Time Energy dissipated Rate of dissipation (m) (MJ) (MW) BICYCLE 8 1.8 s 0.003 0.002 CAR 100 6.4 s 0.73 0.11 TRAM 87 8.3 s 12 1.4 RAILWAY TRAIN 1433 38.2 s 1080 28 JET AIRLINER 1430 32 s 2000 62 SHIP $>$12 minutes Conclusion Whenever it stops or slows down, a high-speed vehicle sheds a great deal of energy, and the energy is usually wasted. In the case of an automobile, whatever its shape, the presence of the road surface profoundly changes the flow pattern. The road cannot pull it down, so if the pothole has steep sides the wheel in fact will lose contact with the surface completely. We shall distinguish three conditions that occur when a vehicle moves over a rough road: The wheels go into jounce simultaneously, a condition sometimes referred to as 'parallel springing' One wheel goes into jounce while the other remains unaffected, assuming that the vehicle body remains upright. It arises from the distribution of stresses within the contact patch. The solution is to make h as small as possible and b as large as possible. But on crowded urban roads, car drivers have little incentive to economise in this way. Such a 'quarter car' model is shown diagrammatically in figure 2, where the two zig-zag lines represent elastic springs (one is the tyre and the other is the suspension spring proper). The wheels themselves are a major source of friction drag. In fact, with the exception of the dead-ahead position, there may be no steering angles at all that produce pure rolling. We shall not go into detail here, but there are two points worth mentioning. A rear spoiler may increase downforce and stabilise vortices to reduce buffeting, but rather than reducing drag, it usually adds to it [9] [22] [29] [31] [40]. In practice, the engineering of a steering system is more difficult than pictured here. But this arrangement fails to meet the first requirement of any steering system, that if the driver lets go of the handwheel, the steering should return automatically to the central position rather than causing the vehicle to swerve unpredictably to one side. An ingenious feature of the original design by Earle S MacPherson is that instead of pivoting on separate body mountings, the fore-aft links on either side are connected via a lateral bar to form an anti-roll bar – hence each link serves two purposes. No mechanical linkage can actually meet this requirement, although several designs come close to doing so. A pinion (gear wheel) on the end of the steering column engages with the rack so that any movement of the handwheel causes the track rod to move from side to side, providing accurate and reliable control (figure 10). But the inertia force can't be abolished as easily as that. Nevertheless the concept was more closely in tune with aerodynamic reality than its predecessors and as such marked a turning point the evolution of vehicle body design. If we allow it to settle to its equilibrium position, the spring will deflect by an amount $$D \quad = \quad \frac{mg}{k}$$ We'll measure any subsequent vertical displacement z of the mass relative to the equilibrium position, positive downwards. Broadly, the vehicle is meant to glide along while the wheels bounce up and down, and not the other way around. Air springs are used on heavy lorries and railway passenger trains. And we shall measure output as vibration delivered not to the passenger's torso, but to the vehicle body at the suspension mounting points, which is not quite the same thing. To generate any given input frequency, the surface profile of the road is pictured as a regular sine wave with peaks rising a distance a metres above the mean and troughs a metres below, that is to say, the input amplitude is a. The pivot axis around which each front wheel swivels, is not, as you might imagine, vertical. Rear suspension Manufacturers didn't need much persuading to switch to independent suspension for the front axle, but the rear axle was a different matter. Here, the feedback provided by the handwheel tells the driver something useful, and in the event that too much is demanded of the tyres and they start to lose grip, the steering will 'loosen up' and perhaps give some warning of what is about to happen. If the body surface is divided in small areas, each with a force acting at right angles to it (figure 11), we see that some of the areas face towards the front, and some towards the rear, but most are inclined at some intermediate angle. Conversely, the axle attachment on the inside spring moves forward. It was an interesting project because the design was based on logical principles throughout. This castor angle, as it is usually called, provides mechanical trail without putting an eccentric load on the suspension. A high ratio has another advantage: at high speeds on public roads, a small input, say, a steering angle of 0.5°, can generate lateral forces that use up all the friction available in the tyres, in other words, the car skids. In principle, an intelligent suspension will anticipate road surface irregularities, lifting the wheel over each bump so that the load remains constant. The drag rises with increasing angle and peaks at around the typical hatchback angle of 30° [36]. Let's make the outer steering angle δ_o vary step-by step over its full range, and plot the inner steering angle δ_i that would be required for zero scrub (the formula used to generate this curve is derived in Appendix 1, equation 27). The results for our single-degree-of-freedom model are shown in figure 6 for a suspension with a damping parameter ζ equal to 0.4, and natural frequency ω equal to 6.28 rad/s (1 Hz). When a vehicle coasts downhill, it is losing potential energy and will continue to do so until it reaches level ground. Parallel and reciprocal springing Front suspension Most early cars had rigid 'beam axles' and leaf springs (figure 15). A similar limitation applies to rail vehicles, and since steel railway wheels running on steel rails have a relatively low coefficient of friction, trains can't stop very quickly and need a relatively large gap between them to avoid collisions. The benefits are greater than they might at first appear. The pressure changes obey Bernoulli's equation, which tells us that when the speed V rises, the pressure p must fall and vice versa [1] [45]. So we'll call our ideal form the 'zeppelin' after the German count who pioneered airship design during the early 20th century. Emergency track brakes for trams Non-friction methods All friction brakes have an abrasive action that removes material from the surfaces in contact, and during the 1970s, an express loco would wear out its brake shoes in about a week. The inside edge of the tread can lift clear of the road surface, which reduces the area of the contact patch and therefore reduces grip. When the driver brakes, a horizontal force is applied to the wheels at road surface level. In the case of a family hatchback, the outline is disrupted by the angles of the bodywork. Any non-zero angle will usually produce an increase in both the drag and the lift, together with lateral forces and yawing couples that can interfere with directional stability. Therefore intensive damping during the upward stroke is undesirable. High-frequency input The next experiment takes place at high speed. If all these are taken into account, modelling of suspension becomes very complicated so suspension designers use computer simulation software to handle the calculations. Then the wheel will oscillate with a vertical displacement given by $$z_{road} \quad = \quad a \sin(\Omega t)$$ The only change compared with the scenario described in the last section is the nature of the input. Second, the pressure distribution on the contact patch becomes lop-sided. A vortex resembles a swirling column or cylinder of air. If the total change in height is h, the change in potential energy is Mgh, and this must be dissipated continually throughout the descent if the vehicle's speed is to be kept under control. The spring can now be compressed, with the load increasing to maximum and then being gradually removed. The rear axle unit and suspension often weigh more than the chassis they support so when the vehicle is empty, the unsprung mass dominates the sprung mass. So much so, that even motoring journalists are apt to complain. The two terms on the right-hand side of equation 5 are therefore modified. Let's start with the pure rolling configuration (equation 11). Tyre scrub with centre point steering The de-stabilising effect of positive offset A negative offset achieves the same goal, with the added advantage that it improves stability under braking. Now for the front wheels. The treatment of the tail was cursory. Conventional configuration When does the traditional configuration work? Secondly, lift at the rear axle may have the effect of increasing the slip angle of the rear tyres relative to the slip angle of the front tyres, which can lead to oversteer and in extreme cases make the car dangerous to drive (see Section C0418). The large output arises because the system accumulates individual pulses of energy and doesn't give them back. The 1 Hz frequency therefore turns out to be a safe 'window'. Separation at the front of the engine cover First, separation can occur at the leading edge of the engine cover. In the case of aircraft, there is almost no separation of the boundary layer so a high proportion of the drag comes from surface friction. First let's look at the fixed parameters – the ones that are built into the chassis of the car and don't change when the wheels are swivelled. For an aircraft, there is little or no separation, so friction drag is important, while pressure drag is not. Hence the car will rotate or yaw away from the direction of the wind (figure 27), thereby accentuating the crabwise deviation. A power steering system senses what the driver wants and replaces the driver's input with a more powerful signal. The error is now much reduced across most of the range, and we also see that for two particular values of δ_o, namely zero and 25°, the correspondence is exact. The wheels must move up and down without throwing the car off course. The tyre slip angles (see Section C1717) become significant, and since load is transferred from the inside to the outside wheel on any curve, the inside and outside slip angles will be different. With a narrow wake and relatively modest negative pressure, the drag is reduced [8] [34] [38]. Figure 1 shows the components of a double wishbone assembly. The complementary function is the same as for equation 6, but it doesn't concern us here, because it decays to zero over time. If there were no agency to prevent it doing so, after going over a bump, the body would continue to bounce up and down indefinitely, shaking the passengers like jelly. So a large reduction ratio reduces any risk arising from an unintended movement of the handwheel. We have seen that when the wheels bounce up and down, the body bounces up and down too, but mostly to a lesser extent. It follows that the interior of the engine compartment represents a significant aerodynamic streamlining problem in the same way as the exterior. The effect of the road surface on air flow around the ideal shape So why not cut it in half? When the wheels of a car roll up a hill and down the other side, the body rises and falls too, and if the road is perfectly smooth there will be no distortion of the springs. Advanced embedding details, examples, and help! You're Reading a Free Preview Pages 6 to 13 are not shown in this preview. This is very different from the traditional configuration, and we'll consider the implications later. Other quantities whose values change with δ_o include the angles α and β together with the length of the diagonal e. The formula enables one to make comparisons between bodies of different shapes and sizes. One can see from figure 3 that $$a \cos (\theta - \delta_{o}) \; - \; s \sin \sigma \; - \; a \cos (\theta + \delta_{i}) \quad = \quad 0$$ Now we look at their lateral projections onto the line OI in figure 3; these three projections sum to w. Phase lag and energy losses Now for the final step. The couple were smiling broadly as well they might, because the car was invisible and they were floating on air. These springs later evolved into the semi-elliptical springs used on early automobiles (figure 5), and heavier versions formed the basis of suspension for steam locomotives and rolling stock. But within a few years, all these considerations could swept aside. The irregular fluctuations in contact pressure between the tyre and the road also compromise roadholding [1]. They are really different ways of adjusting offset. This is only possible with a 'squareback' or estate car design. But on the rear wheels it does exactly the opposite. The components of drag for an aerodynamically efficient car Like the lift force, although drag appears to be a complex amalgam of forces generated in different ways, the overall value acting on a moving vehicle can be estimated using a relatively simple formula. Again, the values of load and deformation will trace out a single curve, this time in the negative region. With the four-bar linkage, this means fixing the point of intersection of the steering arms aft of the rear axle. Finally, the two lever arms are linked by the track rod PQ, whose length is denoted by s. Your hand represents the input, by which is meant the roughness and bumps on the road surface. Indeed, if you blank off the radiator inlet the drag will fall and the car will actually go faster (until the engine overheats). Deflections of either kind will distort the steering linkage and affect the relationship between steering input and output. Rack-and-pinion linkage Fixed parameters for the rack-and-pinion linkage The variables are the outer and inner steering angles δ_{o} and δ_{i}, the angles of rotation ψ and ξ (both positive anticlockwise) of the two steering links, and the lateral motion h of the rack. Also, the angle ϕ can be negative, and so can the setback u, and each of these possibilities leads to a variation that may appear geometrically distinct but in fact obeys the same mathematical equations. The anti-roll bar All these types of suspension are shown reacting to a bump encountered by the left-hand front wheel. Why? Together they handle surface roughness that comes from potholes and other large-scale irregularities in the road surface level, typically hundreds of millimetres across and more. The experiment tells us is that a sprung suspension behaves differently at different input frequencies. The reason is that a horse needs a centripetal force to keep it on a curved path. But the damping effect was not smooth and predictable. Small wheels were once common on cars such as the BMC Mini, but their rolling resistance was high and the tyres wore out quickly. L is the distance between the front and rear axles, usually known as the wheelbase. In the case of a double wishbone suspension for example, the wheel hub pivots around two ball joints. Hence $Q h - \delta P L = 0$ and we observe that the increase in load at the front wheels is given by $$\delta P \quad = \quad Qh/L$$ In fact, without some careful engineering of the suspension, the car will 'dive' or pitch nose-down to some degree. Load - deflection curve for a steel spring But when they are loaded and unloaded very quickly, not all objects behave in this way, and nor does a vehicle suspension unit. Fortunately, on most roads the bumps and hollows are not regularly spaced, so their impacts occur at more-or-less random intervals. This is not necessarily a drawback. Hence with positive camber on a flat road surface, the two front wheels tug in opposite directions as they attempt to roll towards opposite sides of the road. We'll come back to this possibility later, in section C1508. It was this problem that led the designers of early automobiles to develop the 'Ackermann' steering layout, which has survived on motor vehicles in modified form until the present day. When a tube curves around a vehicle body it is squeezed by its neighbours into a smaller cross-section (figure 12). The couple acts in such a way as to tumble the car over on its nose, like a cyclist being pitched over the handlebars. Ship's captains do it because there is no alternative. Yet Buckminster Fuller's famous Dymaxion of 1933 seemed to run quite well [5]. © 1996-2014, Amazon.com, Inc. Hence as shown in figure 6, the cornering force is concentrated at a point roughly three-quarters of the way along the contact patch [6] [8]. BOWDEN, F P and TABOR, D (1974, 2nd ed) Friction: an introduction to tribology London: Heinemann, see p144. Castor trail on a supermarket trolley Restoring force from castor trail Mechanical trail On a supermarket trolley, the pivot axis is vertical, so the two main forces on the suspension - the weight of the body pressing down from above, and the reaction from the road surface pressing up on the wheel from below - are not aligned (figure 4). But it would be rather unwieldy. The need for castor trail Why rear-wheel steering is unstable Rear-steering vehicles are fundamentally unstable. Because the damper exerts a force that continually dissipates energy, the system must decay towards its static equilibrium position, and the value of ζ determines the manner in which this happens. To start with, we must explain two of the parameters that characterise suspension behaviour. The total downwards force acting on the body is equal to its weight minus the upwards thrust $Mg+kz$ from the spring thus: $$Mg \; - \; (Mg\,+\,kz) \quad = \quad -kz$$ Since force equals mass times acceleration we can then write the equation of motion of the mass as $$M \frac{d^{2}z}{dt^2} \quad = \quad -kz$$ which leads to a second-order linear differential equation whose solution (see, for example, [7]) is a sine wave of frequency f_N given by $$f_N \quad = \quad \frac{1}{2 \pi} \sqrt{\frac{k}{M}} \quad \text{Hz}$$ For a vehicle suspension at a wheel that carries a static load P, we can substitute P/g for the mass M in the above equation and predict the natural frequency of the suspension as $$f_N \quad = \quad \frac{1}{2 \pi} \sqrt{\frac{kg}{P}} \quad \text{Hz}$$ Revised 11 February 2015 Simple harmonic motion of a mass attached to a spring Page 4In 1961, the Chevrolet car company commissioned a prize-winning TV advert that showed a young couple riding along in their car; it can still be viewed on the internet [26]. The reactions arising from the vehicle's weight will therefore remain constant. This was common among cars designed during the 1970s, when a sharp leading edge for the engine cover was fashionable (figure 17); fortunately, the drag penalty was modest, because a boundary layer that separates near the leading edge usually re-attaches itself a little way downstream. And of course the forces in the springs themselves vary from moment to moment with every impact from the road surface. It's important to avoid waves in the road surface that inject pulses at frequencies around 1 Hz. But for frequencies above 1.5 Hz, the suspension cuts down the level of vibration appreciably, which is what we want. In what follows we will try to work out the arrangement that comes closest to what was intended. But it can't easily be scaled up to passenger car size and run at passenger car speeds, because when you scale up a model the mass increases rapidly and so does the rate at which energy is fed into the vehicle from bumps in the running surface. We get $$\delta_{ip}'' \quad = \quad \frac{2\left(\frac{w}{L} \right) \cos (2 \delta_{o}) \;\; - \;\; 2 \left(\frac{w}{L} \right)^{2} \sin \delta_{o} \cos \delta_{o}}{\left[1 \; - \; \left(\frac{w}{L} \right) \sin (2 \delta_{o}) \; + \; \left(\frac{w}{L} \right)^{2} \sin^{2} \delta_{o} \right]^2}$$ And evaluating with the steering in its central position in the same manner as before, by putting $\delta_{o} = 0$ in equation 33 we get $$\delta_{ip}''(0) \quad = \quad 2 \left(\frac{w}{L} \right)$$ Appendix 2: The four-bar linkage Relationship between inner and outer steering angles First, we find an expression for the inner steering angle in terms of δ_{o}. Required deceleration rates for different vehicle types Category Vehicle details Mass Initial speed g (tonnes) (km/h) ($\times$ 9.81 m/s2) BICYCLE 0.09 32 0.50 CAR Typical family saloon at motorway speed 1.5 113 0.50 TRAM Midland Metro (Birmingham, UK) [9] 54 75 0.255 RAILWAY TRAIN TGV Paris Sud Ouest [8] 385 270 0.20 JET AIRLINER Airbus 380 [6] 500 322 0.285 SHIP Oil tanker 200 000 25 NA So taking the various constraints into account, how much stopping distance does each type of vehicle need, and how much energy must be dissipated during the process? Imagine the pivot axis extended until it meets the ground at point P as shown for the left-hand front wheel in figure 10. Cheap and rugged, they are still produced for trucks. By contrast, with a negative offset, the wheels swivel in the opposite direction. But a more important reason is that the car handles better. If one of the front wheels, the left-hand wheel say, loses grip on a slippery patch (or if the brake on that wheel fails) the imbalance in the resultant braking force tends to make the whole steering system swivel to the right. A line is projected at right-angles from the centre of each front wheel to pass through a single point X on the projection of the rear axle. Careful design is needed to overcome these effects. Then the body is truncated at the 'natural' point of separation. Roll-induced steering of this kind was quite common fifty years ago, when most cars were equipped with a rigid rear axle suspended on leaf springs, and this arrangement is still commonly found on light vans and trucks today. We shall use a prime symbol in each case to denote differentiation so that, for example, $$\sigma' \quad = \quad \frac{\mathrm{d} \sigma }{\mathrm{d} \delta_o}$$ $$\sigma'' \quad = \quad \frac{\mathrm{d}^{2} \sigma }{\mathrm{d} \delta_{o}^2}$$ Variables for the four-bar linkage The problem As explained earlier (Section C0505), for many decades it was the custom to arrange the steering arms so that their projections intersect at the rear axle (figure 4). Those of us who find speed exciting will want a powerful engine, and perhaps fit aerodynamic 'add-ons' such as a front dam, rear wing, spoiler, or side skirts, with the intention of reducing drag and making the car go faster. You may be puzzled about the blunt nose. Each wheel is mounted on a short stub axle that is pivoted at a kingpin. Since the coefficient of friction between wheels and track is only around 0.25, aerodynamic assistance can be invaluable for emergency braking at this speed. We can now distinguish an internal force by virtue of the fact that it operates entirely inside the envelope, and is not detectable (at least, not directly) from any distortion of the film. The variation arises from the narrowing of the space occupied by each parcel of air as it moves around the vehicle body, which acts as an obstruction. To see this, replace the wheel load P in equation 2 by Mg, square both sides and swap them around to get $$\frac{k}{M} \quad = \quad (2 \pi f_N)^2$$ From this we see that $\omega = 2 \pi f_N$, and it is measured in radians per second. Components of a front suspension Front beam axle with kingpins Most of the joints in a suspension system are slightly flexible, being cushioned with elastomer bearings that absorb shocks and therefore reduce wear as well as doing away with the need for regular lubrication. Because its shape is symmetrical, the pattern of streamlines must be symmetrical too, and if we imagine a horizontal plane slicing through the nose and tail (points O and P in figure 20), the proportion of air flowing over the top of the airship above this plane must be the same as the proportion flowing underneath. The most suitable value for the reduction ratio depends on how you intend to use your car. Firstly, by reducing the contact force between the tyres and the ground, it reduces grip. However, this normal pressure is more complicated than it first appears because it consists of two parts: pressure drag or form drag together with a component that we haven't so far mentioned, the vortex-induced drag that is tied up with trailing vortices. Based on the above figures, table 3 shows for each vehicle type the distance and time required to stop. Common sense suggests that it should, but the picture is a mixed one. Once aloft an aircraft can't stop at all, and even flying slowly is not a good idea. Previous C.2009Next C.0816 There is a well-known saying among automobile engineers that the brakes of a car take out all the energy that the fuel puts in. It is negative because it acts upwards on the quarter-body. Longitudinal complience in a vehicle wheel The problem of unwanted movements Assuming we can get the mechanism right, there is still the problem of distortion under load. We picture the body at displacement z after time t, and work out all the forces acting on it as follows: Weight: the weight of the quarter-body can be pictured as a force Mg acting vertically downward at the axle centreline. The cornering force is reversed, and together with the unbalanced braking force, it produces a resultant force that passes roughly through the vehicle's centre of mass. The position of the track rod almost comes into the same category because its position is constrained by the space available behind the engine. It will soon be possible to build cars with a computer-controlled steering system in which the front wheels swivel independently, driven by electrically-powered actuators. Most surprisingly perhaps, as explained in Section G0119, suspension reduces rolling resistance and makes the vehicle go faster. And less contact means less grip. Some notional deceleration rates are set out in table 2; they represent what is required from the braking system during an emergency stop. If we work out the shear force on each element, then add all the components of shear force aligned against the direction of motion, we obtain the total friction drag that acts on the vehicle to slow it down. In the case of an automobile it's the other way round: the key is to eliminate or postpone separation and to minimise the cross-sectional area of the wake. Although the same principles apply to both, in practice the drag comes from different sources and therefore cars and planes call for different types of aerodynamic treatment [2]. Force x velocity plot for a linear damper Unfortunately, because the damper resists compression, its resistance undermines the role of the springs. Each additional spring introduces an extra degree of freedom in the sense that relative movement can take place independently at each stage. The result is indicated by the green curve in figure 10, which indicates that the error is extremely small for all steering angles up to about 10 degrees. Load - deflection plot for a linear spring The stiffness of the spring, however, is not the same thing as the stiffness of the suspension. Beam axles have the desirable property that they preserve the geometrical relationship between the two wheels: when one wheel goes over a bump (diagram B in figure 14), the track width remains constant so that the tyres do not 'scrub' the road surface laterally, which would accentuate tyre wear. The brakes are flat fins aligned at right-angles to the direction of motion (figure 4). A heavily-loaded car may not be very safe, but it gives a smoother ride. It passes on the motion to the spring-and-damper system, which in turn passes it on to the body but in modified form. Precision is also important for reducing tyre wear. Since that time, governments throughout the world have introduced minimum standards for almost all types of vehicle, and later, we shall see how engineers have responded (brake technology for cars is reviewed in section C0816). Load transfer while cornering Lateral transfer When a car travels round a curve, load is transferred from the inside wheels to those on the outside of the curve (figure 6). The result is shown in figure 5. The process of dealing with road surface irregularities takes place at several different levels within this cascade. The spring is located inboard of the wheel, and connected to the wheel through a series of mechanical links. First, there is friction with the boundary layer that acts like a shear force along the body surface. The reason is that when the suspension is configured with a positive offset, the front wheels encounter rolling resistance, which drags them backwards relative to the car body: they swivel outwards, which tends to wear out the tyres. The snag is that the error rises quite quickly thereafter, showing that the inner wheel swivels too far as the outer wheel approaches full lock. The forces vary among them, each having a greater or lesser component in the direction of motion. Given a drawing board and a blank sheet of paper, how might one construct a layout - any layout - that passes the test? The curve isn't plotted in figure 7, but as you can guess, the opposite effect occurs. Early cars and trains didn't have separate dampers, and didn't need them, because they were mounted on leaf springs. Finally, there may be 'bump stops' that are intended to prevent damage to the chassis, which can itself flex slightly, as of course can the passenger seats. They canter steadily round the ring, leaning towards the centre (figure 1). But unless the atmosphere at ground level is stationary, it will usually be aligned at an angle to the direction of motion (figure 26). Imagine you are cruising down the autobahn at 140 km/h. Body roll ('reciprocal springing' [17]). BIRD A (1973, 2nd ed) Roads and vehicles London: Arrow, see p96. We assume that the left-hand front wheel swivels to the right through an angle δ_o (the subscript o indicates that this wheel is the 'outer' wheel, i.e., furthest away from the centre of the curve). Offset is the lateral distance measured between P and the centre of the contact patch. Nowadays, most cars retain a smaller degree of Ackermann compensation, with the point of intersection of the steering arms aft of the rear axle [14]. This may not seem like a difficult engineering challenge, but at motorway speeds, the effects of small deviations such as a wheel pivoting slightly out of the nominal fore-aft direction are magnified. Incidentally, it is close to the frequency associated with footsteps in steady walking, and perhaps for this reason, it lulls babies to sleep [1]. The second element is the normal pressure that acts at right-angles to the body surface. To find the overall lift force we could in principle divide the body surface into small elements and work out the magnitude and direction of the normal force acting on each element, then sum the components that act at right-angles to the direction of motion. Heavy braking over long periods can heat up the brakes until they no longer perform properly, and it is for this reason that long downhill grades present a further design challenge for road vehicles (see Section C0816) and railway trains. Most cars today are aerodynamically efficient and there is little scope for improving the standard configuration [42]. There two sorts of stress distribution: (a) normal stress, that is to say, pressure at right angles to the road surface, and (b) shear stress, which acts within the plane of contact between the tyre tread and road surface. Here, the speed of the air in the boundary layer rises consistently as it moves around the nose and along the vehicle sides and roof, so the pressure is falling and with local exceptions the boundary layer has no difficulty holding on to the surface - in fact its contribution to pressure drag may be small and in some places negative. By contrast, any external force must be transmitted through the film. We are interested in the way the vehicle bounces when driven steadily over a road with continuous vibration input. It feeds its kinetic energy to the spring until the increasing compression of the spring halts the downward motion. Early attempts to develop a 'springy wheel' for horse-drawn carriages failed, but a WW1 aircraft, the Linke-Hofmann RII, had steel wheels each of whose wooden tyres incorporated a multi-spiral spring [24]. At each corner, the wheel arches were cut out with the smallest possible area to discourage the air flow from interaction with the wheels. Effect of roll on outer rear leaf spring Roll-induced steer Improved design with canted and flattened spring profile Camber change Although shimmy has been eliminated, the gyroscopic forces can still cause difficulties. The effect of reduced damping How does the picture change if we alter the suspension characteristics? It is lowered gradually until it settles into its equilibrium position. At another level, it creates forces that interfere with the car's motion. Each front-wheel may be carrying a load approaching half a tonne. During the 1920s, what mattered was a large hood and radiator grille because they signalled a powerful engine. All are shown in figure 13. There are six of them: L, w, a, θ, u, and s, as shown in figure 2. The force is supplied by friction acting laterally on each of its hooves. But the imaginary line joining them defines the axis of rotation, and this line is still referred to as the kingpin axis. Conclusion So with carefully tuned dampers and lightweight suspension, are we any closer to the designer's dream of a vehicle floating along like a cloud while the wheels and suspension rise and fall over the road surface roughness beneath? The larger the surface area of the vehicle skin, the greater the drag. As you might expect, the gain is largest around the natural frequency, and this is critical for road construction. It turns out that the sensitive frequencies are all higher than 1 Hz, except the one associated with motion sickness, which does not injure passengers even though it makes them feel ill. Feedback An irreversible mechanism ensures that when a front wheel bashes into a pothole, the driver doesn't receive a kick from the handwheel. During the compression stroke, successive points trace a path up the left-hand side of the ellipse, while during the rebound stroke the path returns down the right-hand side. The reason we avoided inertial forces when describing the circus horse is that they don't actually exist. The most common type is the hydraulic damper (figure 11). This represents the ideal case. The braking distances for the high speed train and for the passenger airliner are both well over a kilometre. Results for the case where a second 'spring' represents the tyre can be found in [5]. The motion of the linkage is graceful but curiously, the mathematics is not. Negative trail Not all vehicles are configured with the pivot axis raked astern [1]. We'll return to this matter later in section C0504. A small amount of camber – either positive or negative - puts a load on the links that connect the front wheels with the steering assembly. The two curves don't seem very different, but if we plot the percentage error in δ_i, that is to say, the percentage difference between the angle needed for pure rolling and what the traditional 4-bar set-up actually produces, we see that it succeeds only when the steering is in the dead ahead position ($\delta_{o} = \delta_{i} = 0$). At this point, the body begins to rise again and the cycle is repeated. There are several reasons for this, some of which have their roots in the nineteenth century when early car engineers looked to the horse-drawn carriage as their model. Hence if we take moments about the centre of mass, these forces don't appear in the equation. As it rises, in a sense it collides with the vehicle body, passing on momentum through the suspension. Hence we need the particular integral, which takes the form: $$z \quad = \quad {C_1} \sin (\Omega t) \;\; + \;\; {C_2}\cos (\Omega t)$$ where C_1 and C_2 are arbitrary constants. Sooner or later, manufacturers will introduce 'drive-by-wire' in which there is no mechanical linkage between handwheel and steering arms at all. So can we apply zeppelin shape to build a really efficient automobile? If we take the lift coefficient as 0.4 and the density ρ of air as 1.226 kg/m3 (this is strictly speaking the value at sea level [3]) then when cruising at motorway speed, say 120 km/h or 33.3 m/s, the lift force comes to $$0.5 \times 1.226 \times 33.3^2 \times 0.4 \quad \times 2.0 = \quad 544 \; \text{newtons}$$ which is equivalent to roughly 50 kg or a little under the weight of an adult passenger. Retarding forces under braking However, there is another way to stop a railway vehicle: instead of gripping the wheels, the brake pads can grip the track instead. The faster the car is travelling round a curve, the greater the restoring force, and this provides useful feedback for the driver. It turns out that the 'base pressure' within the wake is roughly uniform across the whole of its cross-section. Aerodynamic theory together with wind tunnel tests have confirmed that the drag coefficient for such a shape is in the region of 0.07 to 0.09 [44] [46]. If you have ever built a model car with front wheels that swivel exactly in parallel, you'll have noticed how they grind across the table top during sharp turns. Firstly, it creates a lateral force that can be pictured as acting at a single point - the centre of lateral pressure. To prevent this, the engineer must interpose between the driver's brake pedal and the four brake disks a system for balancing the distribution of braking effort in proportion to the instantaneous wheel loads, which requires some fairly sophisticated engineering (Section C0816). TUPLIN, W A (1980, 2nd ed) The steam locomotive: its form and function Bradford-on-Avon: Moonraker, see p60. The impacts of the molecules act on the vehicle surface in two distinct ways. An opposite effect occurs during acceleration, when the load shifts towards the rear axle and the vehicle tends to 'squat' tail-down. For over 100 years, the geometry of the steering linkage used by most car manufacturers was based on a simple rule of thumb. Ideally, we want the wheel to remain in the same vertical plane throughout so that it doesn't crab sideways, or swivel out of alignment, or tilt away from the vertical (figure 6). The force that actually slows the vehicle down comes from the road surface acting on the tyre tread (figure 1). The handwheel Nowadays we expect the front wheels of a car to swivel easily from lock to lock so we don't have to wrestle with the handwheel while manoeuvring into a cramped parking space. When the streamlines are squeezed together at the highest point of the arch, the pressure is appreciably reduced, and produces lift just like the wing of an aircraft although the motion of the air is messier and geometrically more complicated. First and second differentials Now we find expressions for the first and second differentials of the inner steering angle evaluated for the central steering position with $\delta_{o} = 0$. We want both tyres to roll smoothly without scrubbing sideways across the road, otherwise they will wear out quickly and the vehicle may not handle very well. The more extreme the manoeuvre, the greater the reaction. At no time does the rubber band stretch or contract noticeably. For example, a tyre tread surrounds local irregularities in the road surface such as the sharp edge of a pothole or coarse particles of aggregate projecting from an asphalt road surface. From the diagram, we see that $$\cot \delta_{o} \quad = \quad \frac{R \; + \; \frac{1}{2}w}{L}$$ $$\cot \delta_{ip} \quad = \quad \frac{R \; - \; \frac{1}{2}w}{L}$$ Subtracting equation 26 from equation 25 leads to the following relationship between the two steering angles: $$\cot \delta_{ip} \quad = \quad \cot \delta_{o} \;\; - \;\; \frac{w}{L}$$ Later on, we shall need the first and second differentials of δ_{ip} with respect to δ_{o}, in other words the quantities $$\delta_{ip}' \quad = \quad \frac{\mathrm{d} \delta_{ip}}{\mathrm{d} \delta_{o}}$$ $$\delta_{ip}'' \quad = \quad \frac{\mathrm{d}^{2} \delta_{ip}}{\mathrm{d} \delta_{o}^2}$$ The easiest way to proceed is to differentiate both sides of equation 27 to get $$\text{cosec}^{2} \delta_{ip}.\delta_{ip}' \quad = \quad \text{cosec}^{2} \delta_{o}$$ If we now substitute $1 \; + \; \cot^{2} \delta_{ip}$ for $\text{cosec}^{2} \delta_{ip}$ and in turn substitute the right-hand side of equation 27 for $\cot \delta_{ip}$, after simplification and rearrangement we get $$\delta_{ip}' \quad = \quad \frac{1}{1 \; - \; \frac{w}{L} \sin(2 \delta _{o}) \; + \; \left(\frac{w}{L} \right)^{2} \sin^{2} \delta_{o}}$$ This is an explicit expression for the rate of change of δ_{ip} with δ_{o}. Static deflection of a vehicle wheel Static deflection Imagine a car hanging from a crane with its wheels just touching the ground (figure 1). On the other hand, the axle tilts, and gyroscopic precession gives the steering a kick. We shall also need to differentiate some of these variables with respect to δ_o. The association with aircraft was important: 'streamlined' body contours suggested minimal resistance and greater speed. Air passengers are also strapped in, and are accustomed to moderately large g-forces on landing. A useful overview of lorry steering can be found in [13]. We therefore speak of load transfers among the wheels. Apart from the question of wear and maintenance, they were quieter than friction brakes and the braking effort was easier to control. or its affiliates Loading PreviewSorry, preview is currently unavailable. Looking at the middle diagram, let us imagine first removing mass and then lowering it onto the spring. The overall drag force D, measured in newtons, is given by: $$D \quad = \quad \frac{1}{2} \rho V^{2} C_{D} A$$ where C_{D} is the drag coefficient, and as before, ρ is the density of air, V is the wind speed relative to the vehicle, and A is the cross-sectional area of the vehicle body. In principle, an all-enclosing envelope ought to reduce air resistance, but neither were very efficient, and styling conventions sometimes made matters worse. In effect, the driver is jacking up the car [12]. Variables for the rack-and-pinion linkage In principle, this configuration can be analysed in the same way as the traditional four-bar linkage, although the geometry is more complicated and the author has not been able to derive a formula for the inner steering angle δ_{i} explicitly in terms of the outer steering angle δ_{o}. The intention was to minimise tyre scrub, in other words, to make each front wheel roll in the direction it is pointing without any sideways scrubbing of the contact patch. On early vehicles a kingpin was fixed to either end of the beam axle and each wheel hub pivoted around bearings on the corresponding kingpin (figure 2). When its value is less than one, the system is said to be under-damped. The old-fashioned beam axle is the only suspension that keeps the wheels both perpendicular to the road surface and at constant track while the body tilts. Bumps that 'kick' the steering are generally undesirable, not least because any such reaction knocks the wheel off line and changes the direction of the vehicle. Privacy policyCookie settings Again we imagine the body surface divided into small areas, each with a shear force acting on it. We'll take θ as the 'optional' parameter. Having proved themselves as back-up devices, rheostatic brakes then took over as the principal system on British locos, where they were designed to dissipate energy at a rate of 2 MW [5]. Different parts of the human body such as the head and stomach are sensitive to vibration at different frequencies (see Section G0216), and can be damaged if the input exceeds a certain level. In practice, this 'ideal' arrangement would only be considered for a slow-moving vehicle such as a mobility scooter. Because a car with front-wheel drive (FWD) doesn't drag its wheels along, but rather, the wheels drag the car, and logically, a stable FWD configuration requires negative trail. The result is shown in figure 9. If a wheel leans outwards on a curve, the tyre slip angle tends to increase and worse, part of the tread may lift clear of the road surface and lose grip (see Section C1717). Reduction ratio The effort can be reduced by increasing the gearing ratio built into the steering system. Therefore a blunt leading edge is as good as a sharp one, and moreover it has practical advantages because although not ideal, a rounded front is better suited than a pointed one to accommodate passengers or a bulky cargo. We know that in the central steering position, both steering angles are zero, so δ_{ip} (0) = 0. Early landmarks in aerodynamic design Early car constructors didn't worry much about aerodynamic design because their machines didn't go fast enough for the air resistance to matter. By calculating the gain we can see how the suspension has damped or amplified the waves. In this respect, the car designer faces an interesting problem. Since the passengers are strapped in, a deceleration of $0.5g$ is tolerable. A torsion spring is just a steel rod that resists being twisted, whereas a coil spring is really a torsion spring wound into a helix (figure 6). The geometry of a rack-and-pinion steering system is defined by six independent parameters. Dampers That is why a suspension system requires some sort of damper, whose role is (a) to dissipate the energy of vibration after a shock load, and (b) to prevent the build-up of resonance. In fact, it is tilted both fore-and-aft as well as from side-to-side, and there are several other ways in which the front-wheel set-up differs from what you might expect. In order to persuade the inner wheel to swivel further we need to increase the angle θ. But a soft suspension isn't good for road-holding because the car lurches under heavy braking or acceleration, and drivers who want to go fast are deprived of feedback from the steering. What about the driver, who is responsible for keeping the vehicle firmly on the road? As we saw in section C0505, the wheel on the inside of the curve must swivel further than the wheel on the outside because it is travelling along a smaller radius. Spring construction When iron-masters began making steel during the seventeenth century, coachbuilders found it a useful material for laminated springs. By contrast, low-frequency excitations of the order of 1 Hz are passed further up the chain to the suspension spring proper, working in parallel with the damper. The drawback is that the amount of spring travel available for soaking up road surface roughness decreases as the cars fills up with passengers, until the car is finally resting on its bump stops [12]. This applies equally to braking and cornering manoeuvres. At displacement z, the compression force in the spring is equal to $k(D+z)$, which if we substitute for D via equation 20comes to $Mg+kz$. The trend nowadays is to retain the larger wheel size but to cast the wheel disc from a more sophisticated, lighter material such as aluminium or magnesium alloy. Those who could tolerate the motion were said to have been 'coached'. The resulting torque doesn't slow the vehicle directly. This force will push the vehicle sideways across the road to a greater a lesser amount depending on the tyre slip angle. The best results are achieved with a very shallow slope of 15°to the horizontal (usually referred to as a 'fastback'). Rheostatic railway brakes Our third and last category relates to aerodynamic devices that grip neither the wheel nor the track, but the air through which the vehicle is moving. Wing mirrors and the requirement for clearance within the wheel arches push it up to 0.3 [11]. Comparison of brakes across transport modes Material Coefficient of friction Lifetime Automobiles [7] Resin-impregnated fibre pad on cast iron disk 0.3 – 0.5 Several years depending on use Railway vehicles [3] Cast iron shoe on steel wheel rim 0.2 1 year Aircraft landing gear [4] Sintered metal 0.3 Several dozen applications Aircraft landing gear and high-performance cars Carbon fibre in ceramic matrix 0.4 - 0.6 Depends on usage Notice that the coefficient of friction μ_{s} rarely exceeds 0.4 (we are referring to sliding friction here, hence the subscript s). The problem receded with the arrival of wider tyres, independent front suspension, and dampers attached to the steering mechanism. In this sense the driver will lose all direct contact with the road outside, and cues that previously arrived through the handwheel will be replaced through artificially-induced feedback. In order to maximise grip, suspension designers aim for a higher slip angle on the outside tyre, which means a reduced degree of Ackermann compensation. In fact, if the wheel is too far ahead, any deviation from the straight ahead position tends to increase of its own accord, and like the castor of a supermarket trolley, the wheel will try to flip round 180° and tear itself off the car. Beam axle with leaf springs Modern independent suspensions deal with precessional effects by (a) removing the connection between the front wheels (hence the origin of the term independent suspension), and (b) by providing a system of links that constrains each wheel to move up and down more-or-less within its own plane. How did the designers avoid instability? The result is vehicle body vibration: the quarter-body moves up and down echoing the movements of the wheels, but not necessarily in phase or by the same amount. The aerodynamically efficient automobile The shape of an automobile is determined primarily by the need to carry people who are sitting upright between four wheels. The aim is to keep it stationary, or at least, to limit the amount it bounces up and down. Most springs today are not 'linear' but 'progressive', in the sense that a progressively larger force increment is needed to

produce a given deflection increment as the vehicle sinks further on its springs. All cars are under-damped, with the damping parameter ζtypically fixed at around 0.4. Free oscillatons of a quarter-car body figure 5 shows a specific response curve for a quarter-vehicle of mass 400 kg having a spring stiffness of 20 kN/m and damping coefficient 2000 Ns/m. But in order to see which frequencies are likely to cause problems, we can calculate the response separately for each frequency of interest and plot the results on a graph. High-frequency input is handled by a conventional spring and damper system. This information, together with the static wheel load P, essentially fixes the suspension stiffness regardless of any other requirements. The zeppelin skin was then stretched over the car chassis with the smallest possible surface area to minimise aerodynamic friction. There were two main varieties, each with its own characteristic frequency, the higher one being associated with the gyroscopic properties of the spinning front wheels. As in the previous analysis, we treat each of the variables δ_{i}, ψ, ξ, and h as functions of δ_{o}. Hence the whole axle deviates out of line, effectively steering the vehicle more sharply into the curve (figure 9). Each wheel on the landing gear handles a braking load equivalent to the 8 wheels of a moderately heavy truck, and multiple disks are needed to shed the heat into the surrounding air so it doesn't set fire to the tyre. The quarter-car model We've simplified the suspension model by ignoring any flexibility in the links and bushes, and in order to work out the natural frequency, we treated a single wheel supporting one corner of the car as if its behaviour could be considered in isolation from the rest of the vehicle. When the driver turns the handwheel towards full lock, each wheel descends relative to the wheel arch (figure 11). The kinetic energy embodied in a vehicle of mass M travelling at speed V is $\frac{1}{2}MV^{2}$, and to make the vehicle stop, all of this energy must be removed in a very short time. By contrast, an offset equal to (say) 25% of the width of the contact patch will allow the tyre to roll around a small arc when the handwheel is turned, even when the car is at rest. An increase in the steering angle in turn causes the vehicle to corner more sharply, which leads to a greater centripetal force, and so on (figure 12). By contrast, a power-assisted system works in parallel with the mechanical system so that the mechanical link is not broken. The second reason is that passengers are bouncy too. They cannot 'feel' the road. Forced oscillations So far, we have pictured the vehicle standing still, vibrating only as the result of a single impulse or blow to the body. Hence the driver must work harder to turn the handwheel when steering round a curve – the steering becomes 'heavy'. The angle of a wheel at any given moment is known as the steering angle. Otherwise it falls short across the whole range (figure 6). An electrical signal from the driver's handwheel will be interpreted by a microprocessor that modifies the steering angles to take into account the speed of the car, the radius of turn, and the wheel loads at the time. The suspension springs of the WW1 Linke-Hofmann biplane landing gear were built into the wheels. As shown in figure 4, the slope of the load-deflection curve increases under load. Revised 12 February 2015Page 3Everyone knows what is meant by a 'smooth ride'. But when moving along the road, it receives a continuous input of vibration from the wheels. The body rises too, less sharply than it would have done had there been no spring to moderate the impact. The engine cover should slope steeply upwards from its leading edge, with the windshield raked astern so that the angle between them is as close as possible to 180 degrees (figure 22). Normally retracted inside the body shell, they are extended by hydraulic rams for emergency braking [10]. Let's think about a motor car for a moment. So can the shear stress. All three are shown in figure 14, and we shall refer to them again shortly. Our model framework is the single degree-of-freedom quarter-car model as shown in figure 2 but with the tyre 'spring' removed; we now have just one spring and one damper in parallel. On a modern passenger car, the opposite is true. Skin friction acting on the upper body normally accounts for about 10% of the drag of a family car but it can be more for long vehicles such as buses or trucks [48]. Together they exert a couple on the suspension, and if this configuration were used on a truck with a beam axle, for example, they would twist the axle on its mountings. Carrying the notion to its logical conclusion, one can imagine the springs incorporated within the wheel disc so that only the rim and tyre are included within the unsprung mass. The force in the spring is usually greater than the load on the wheel itself, and the deflection of the spring as the wheel travels say 20 mm upwards over a bump will usually be less than 20 mm (figure 10). The fenders that housed the front wheels tapered gracefully along either side of the passenger cabin, later evolving into something more like the wheel-covers on a high-performance aircraft (figure 5). There are several reasons for this: (a) air flow inside the engine compartment is turbulent which implies loss of energy, (b) some of the energy is dissipated in friction between the flow of cooling air and the comparatively large surface area of the engine and radiator, and (c) as it leaves the engine compartment, cooling air interferes with the external air flow to produce separation and turbulence [17] [41]. The spring lengthens, and then the body falls too. In most cases, with the top folded down the air flow over the passenger compartment is turbulent, so the vehicle as a whole produces as much drag as a typical family saloon. Parameters and notation To make a start, we need to attach symbols to the various lengths and angles that define the movement of the linkage. As we saw earlier, the total of the normal contact forces remains fixed and equal to the weight of the vehicle provided it is travelling on a level road. Similarly, the steering angle of the inner wheel must be larger. In practice, it is easier (a) to tilt the wheel, or (b) incline the pivot axis, or (c) both. Suspension stiffness at the wheel Given a suspension that is soft and compliant, capable of expanding and contracting over a sufficiently large travel or stroke, one might imagine that the vehicle would float along as if the running surface were perfectly smooth. The drag coefficient expresses the effect of vehicle shape independently of any other factor, so that the C_{D} value for a car of a given shape remains the same if the car is doubled in size, for example. They don't look at all similar. The traditional layout demands that the projections of the two lever arms intersect at the rear axle, so we have $$\tan \theta \quad = \quad \frac{w}{2L} \quad = \quad \frac{2.5}{2 \; \times \; 5.0} \quad = \quad 0.25$$ from which we see that θ is about 16.7°. Springs do not eliminate vibrational input: they re-schedule it. Effectively it becomes a different shape having a larger drag coefficient [43]. Typically, the front wheels must be capable of turning through an angle of $\pm 40^{\circ}$ from left lock to right lock [11], while the handwheel is turned through several complete revolutions. There were other problems too, but with the aid of mathematics (see Section G1115) and a great deal of trial-and-error, engineers solved them one by one. When the wheel climbs up a hill and rolls down the other side, the body climbs with it, tracing out an identical path. But for a front-wheel drive car the load transfer is a nuisance: no matter how powerful the engine, when you accelerate, some of the load shifts to the rear axle and grip is reduced at the driven front wheels. We can demonstrate this for a car with wheel base 5.0 m, track width 2.5 m, and u = 250 mm. Moreover, they don't overheat and they don't wear out. This allows the use of softer springs without increasing body roll [7]. The transfer can take place between the front and rear axles, or from one side of the vehicle to the other, or a mixture of the two. One of the difficulties is fundamental: high frequency operation would require the actuator at each wheel to raise and lower an unsprung mass of 45 kg around twenty times per second. Individually, they can be either positive or negative, but their total must be zero. It's not just the smooth upper body panels that count. It doesn't fit naturally inside an idealised aerodynamic envelope such as the half-zeppelin. It was obtained by calculating the parameters ω and ζ via equation 9 and equation 10, which came to 7.07 rad/s and 0.35 respectively. The case for sea-going craft is different again: there's no equivalent of the disk brake for stopping a ship, at least not a large one like an oil tanker. Imagine a miniature airship of roughly the same height and length as a family saloon, in other words, its length is equal to roughly three times its diameter, as shown in the upper part of figure 20. Since mechanical work has to be put into a generator to create a current, the motor then acts as a brake. It is possible to run a car with zero trail or even a small negative trail with the wheel ahead of the pivot axis. In turn, this causes the 'inner' wheel to swivel to the right through an angle δ_i as shown in figure 3. Nevertheless the car performed well on the racing circuit and had a long and successful career almost without modification until the company ceased to produce racing machines. In theory, a steering linkage that matched these curves exactly would eliminate tyre scrub, and the front wheels would have perfect Ackermann compensation. Let's try 21.5°. The only question will be, what happens when the power fails? A second location is the base of the windshield, then over the top of the windshield at the leading edge of the roof. Centrifugal force is just another variety of inertia force, distinguished by acting sideways rather than fore-and-aft. The dashed line slopes up to the right at roughly 45°, showing the inner wheel must turn more-or-less in unison with the outer wheel. Hotchkiss rear suspension layout Semi-trailing arm rear suspension Postscript Using one of the construction kits still sold in department stores, any bright child can make a toy vehicle that rolls smoothly along the kitchen floor. A tubular telescopic device, on a car it is often located inside the coil spring itself, an arrangement whose compactness is helpful given the competition for space inside the front wheel arch (see Section C1405). They are: $$\delta _{i}'(0) \quad = \quad 1$$ $$\delta _{i}''(0) \quad = \quad \frac{2 w \tan \theta }{s}$$ Substituting the results into equation 12 gives $$\delta_{i} \quad = \quad \delta_{o} \;\; + \;\; \left(\frac{w}{s} \tan \theta \right) \delta_{o}^2 \;\; + \;\; \text{terms in powers of } \delta_{o}^{3} \text{ and above}$$ Comparing equation 15 and equation 18, we can now see that the four-bar linkage will give asymptotically pure rolling for small steering angles when $$\frac{w}{L} \quad = \quad \frac{w}{s} \tan \theta$$ or $$\tan \theta \quad = \quad \frac{s}{L}$$ figure 9 shows the resulting geometry: for asymptotically pure rolling, the point of intersection of the steering arms should be located a distance $\frac{1}{2} L + u$ behind the front axle. Pressure drag dominates because a long 'aerodynamic' tail is not practicable, and the largest single component of aerodynamic drag comes from the tubulent wake. This reduced their unsprung weight. This has not always been so: some early cars were fitted with rear wheel steering with the front wheels permanently fixed in the ahead position. Asymptotically pure rolling The geometrical conditions for pure rolling One way to get asymptotically pure rolling is to compare the two equations for the inner steering angle (pure rolling and actual) and see if we can adjust the parameters so that the two equations give the same answer. Accordingly, it is sometimes said, the front wheels of early cars were made to lean outwards so they would be perpendicular to the road surface [4] [7]. We will use it elsewhere when dealing with more complicated phenomena such as vehicle body roll. The heavier the wheel, the more pronounced are the effects of this wheelhop on the quality of the ride, a point that can be appreciated more easily if we imagine what happens when the wheel hits a bump. Otherwise, as shown in figure 7, whenever the wheel meets an obstruction or passes over a bump, it will be dragged rearward relative to the vehicle, twisting the wheel and axle assembly out of line and imparting a 'kick' to the steering. To get some idea of what this entails, lift your armchair and try shaking it up and down with your bare hands. Racing drivers prefer a smaller value so they can carry out spectacular manoeuvres with relatively small hand movements. In practice, a suspension consists of many elements that interact in a complex way so that the wheel can move up and down relative to the vehicle body. Unsprung mass We have already referred to unsprung mass in Section G1114. If nothing else, the d'Alembert force can help by reducing static forces and dynamic accelerations into the same units so they can be plotted together on a force diagram (figure 3). Nevertheless, the analysis throws light on the evolution of engineering practice during the early days of the motor car. 'Output' is the vibration felt by the passengers, which we want to minimise. The price for this downforce is extra drag, and in fact, Formula 1 cars are among the least 'streamlined' of all modern vehicles, with drag coefficients much larger than those of a typical family saloon. This happens for exmple when it hits a pothole. On US cars, the suspension stiffness k for each front each wheel is typically about 25 kN/m (140 lb/in), and for each rear wheel 17.5 kN/m (100 lb/in) [3]. In reality, the air flow is three-dimensional, with a messier and more complex structure than figure 1 might suggest. The unsprung mass includes any component of the system attached directly to the wheel, such as the axle, the brake disk, and the outboard end of each suspension link. Denote the height of the road surface at any point relative to its mean datum by z_{road}. This represents the target against which a modern production car should be measured. If the pressure rise is gradual, energy from the neighbouring mass of air can be fed into the boundary layer via an exchange of fast-moving molecules at a rate sufficient to prevent separation. As you might guess, a high stiffness indicates a spring that resists being stretched or compressed, whereas a low stiffness indicates a spring that deforms easily. A quarter-car model A rail bogie suspension model Suspension performance We are now in a position to model the process mathematically, with damping included. Rather like the rigging of a nineteenth century sailing ship, front wheel suspension has evolved over many years through a process of trial-and-error. Laminated springs Torsion spring and coil spring Today there is a wide variety of materials to choose from, including rubber (figure 7) and compressed air (figure 8). Finally, a large steering ratio reduces the likelihood of the handwheel being wrenched out of the driver's hands when one of the wheels hits a pothole. Neither does it vary significantly with speed over the normal range of variation characteristic of road vehicles [4]. The figure of 62 MW for the Airbus A380 is equivalent to power output of a small electric power station. The inertia force required to account for any particular acceleration must act in the opposite direction to the acceleration with a value equal to the numerical value of the acceleration times the mass. This may not matter for a truck because the sprung mass increases many times over when it is loaded, but it does matter for cars. Having seen broadly how a suspension works in Section G1114, we can now try to quantify the motion of the system and its effect on the passengers. Since the tyre is not included, the model doesn't take into account 'wheelhop' vibrations associated with the unsprung mass. Starting during the mid 1980s, a systematic approach along these lines resulted in several production models with a drag coefficient of about 0.3. One of the first to go into production was the 1982 Audi 100 [21]. Aerodynamic forces All aerodynamic forces act on a vehicle through boundary layer, the thin layer of air closest to the body surface. Longitudinal steel shoes are mounted on either side of each bogie, and in normal operation are held in a raised position just above the track (figure 2). It must be quite a stubby fin, thick enough to accommodate a passenger cabin. The inlet duct for most models is now located at the front stagnation point low down beneath the registration plate where the air pressure is highest [28]. Note that of all the eight parameters listed, a and b are redundant, since the geometry is completely defined by the other six. Note that this angle is measured not along the slope of the glass itself, but as shown in figure 25, from the trailing edge of the roof to the trailing edge of the trunk (boot lid) [33]. What do you see? The shape of the front is relatively unimportant. The teacup and rubber band experiment with low-frequency input Let's begin with a preliminary test. The loud crash you hear when the pick-up passes over the bump is the sound made by the impact of the suspension on the chassis: any tools or oddments lying loose in the back may well fly into the air. Like a bloodhound, the nose should be close to the ground. Damper force: the damper exerts a force equal to the damping coefficient times speed, in other words $-c\frac{dz}{dt}$. Such variation is not unusual. The track width of a family car can increase by 12 mm when the car is fully loaded, compared with the value when empty [22]. All would be well if it returned to its original position and stopped there, but the body has now acquired kinetic energy and continues onward. Effectively, undulations in the road such as a hump-backed bridge are magnified. In figure 7, the second, grey curve represents what happens if the damping is halved. In principle, the clingfilm could be made from a photoelastic material that changes colour when squeezed or pulled apart. Unfortunately, for most saloon cars the side force acts at a point well forward of the neutral steer point, about one third of the car's length from the front [18]. In its simplest form, the damper is filled with oil. It was followed by the ubiquitous hatchback. The supporting chassis carried the passenger cabin together with the engine compartment, wheels and fenders (mudguards) almost as separate entities. Sources of drag for automobiles Now let us look more closely at the individual parts of an automobile that produce drag. Both types of vortex absorb energy and contribute to drag by altering the normal pressure at locations on the vehicle body near to the point at which the vortex is formed. This takes up any slackness arising from loose or worn joints, for example, so that the steering does not meander aimlessly from side to side [7]. For reasons explained in Section C1115, the situation is different on the downward stroke, and more intensive damping can be applied to dissipate energy stored in the spring. As with the horse's hooves, the wheels are too low down to deliver the force where it's needed. Most friction brakes work by gripping a wheel or an axle to slow down the rate of rotation. Aircraft pilots manage even longer descents by other means which we'll touch on elsewhere. Effects of wind yaw So far, we have assumed that the stream of air that resists the car's motion is coming from straight ahead. By definition, the centre of mass is the point at which the horse's weight - together with any inertia forces that might influence its motion – act as though it was located. For example, under the strain of heavy acceleration the front wheels of a well-known family saloon move forwards relative to the car body by a distance of around 14 mm. Figure 3: Image Public Domain, available at Jamais_contente.jpg#mediaviewer/File:Jamais_contente.jpg, accessed 01 August 2021. Then as shown in the Appendix, the value of f_N is given by the formula $$f_N\quad = \quad \; \left(\frac{1}{2\pi} \right) \sqrt{\frac{kg}{P}} \quad \text{Hz}$$ in units of hertz, i.e., cycles per second. Nowadays, high performance brake disks are made from carbon composites, typically carbon fibre in a ceramic matrix. Also in the table are assumed figures for the vehicle mass and initial speed. Components of suspension The main parts of a car suspension are the wheel, the wheel carrier or hub, the spring, the damper, and the mechanical linkage that connects the wheel hub to the car body. Under power, the front wheels drag the car behind them, and therefore swivel inwards. On a dry surface, in theory a car can stop at $1g$, but drivers rarely use the full braking power available even in an emergency. The boundary layer exerts shear forces on every element of the vehicle body surface Secondly, each impact has a component acting at right-angles to the surface, and the cumulative effect is a normal pressure that like the skin friction varies everywhere in magnitude around the envelope (figure 11). But none of the suspension components are entirely rigid; they all expand or contract fractionally when squeezed or stretched, and as explained earlier, elastomer cushions at the joints allow some relative movement. The starting point for aerodynamicist Malcolm Sayer was the classical zeppelin shape to minimise air resistance or 'drag'. First, a higher value would allow some of the vibration to filter through, because a spring system only absorbs frequencies higher than its natural frequency. Since there will be no steering column and no track rod, the handwheel unit can be bolted to any suitable surface inside the passenger cabin. But bumps are not the only motions that matter. Each successive pulse, like a mother pushing her child on a swing, injects energy in the system. By contrast, train passengers might be sitting, standing, or walking about on board at any time during the journey, and therefore likely to fall over if subjected to braking forces of a comparable level. Since the area under any segment of the curve is equal to force times distance moved, each segment can be interpreted as an energy flow: the jounce stroke puts energy into the suspension, and the rebound stroke takes it out. Nevertheless, the results show broadly how to manipulate the properties of the spring and damper to improve the quality of the ride. Or if you prefer, the tyre behaves like a cone rolling around its apex (see Figure 13 in Section C1114). If we wanted to, we could use equation 35 to substitute for e in equation 36 and equation 37, and in turn substitute for α and β in equation 38 to get an equation with δ_{i} on the left hand side and a function of δ_{o} on the right. When the peak cornering force is reached, the self-aligning torque falls to zero and the handwheel becomes slack in the driver's hands as the tyre skids [5]. It accounts for a substantial part of the overall air resistance. All that one can do is tailor the spring/damper combination to give an acceptable compromise, and we'll explore the principles in Section G1115. Hence the tyre filters out frequencies that are too high for the main spring and damper system to deal with efficiently, usually in the range 20 – 400Hz [2]. Aerodynamic railway brakes Some performance comparisons Brake materials Friction brakes rely on sophisticated materials that can transmit frictional forces and conduct heat away from the contact area without melting or changing shape. Since the early 1930s, researchers have known that the tail of the zeppelin can be removed without incurring a large drag penalty provided there is no abrupt rise in air pressure in the boundary layer as the air flows over the roof and around the sides. The values of $\delta_{i}'(0)$ and $\delta_{i}''(0)$ are derived in Appendix 2, equation 45 and equation 46. Independent suspension also has the effect of reducing the unsprung mass, and in the case of passenger cars, by dispensing with a beam axle it allows more room for the engine to be positioned between the front wheels. In an ideal world, the person in control of the vehicle would anticipate events well in advance, and instead of applying the brakes, use the various forms of resistance to motion such as rolling resistance and aerodynamic drag to slow the vehicle down. Revised 12 February 2015Previous C.2009Next C.0816GeneralRoadRailFluid FlowMarineAerospace BERTRAM, V (2000) Practical ship hydrodynamics Butterworth-Heinemann, see p151, 164. Let's begin with castor. The track rod is split into three sections. Start by raising your 'input' hand a few centimetres, and then return it to its original position, the whole cycle spread out over several seconds (see figure 2). There is no fixed angle that can deal with both the power-on and power-off conditions. The cross-section together with the radiator intake became an ellipse, and the corners were faired over the wheels and around the driver's upper body. In principle P could be located inboard of the contact centre (positive offset), or outboard of the contact centre (negative offset) or exactly at the contact centre (zero offset). With an all-embracing body shell like an upturned bathtub that almost hid the wheels, they seemed the epitome of aerodynamic refinement (figure 7). If the load-deflection curve of the spring is a straight line, its stiffness k is equal to the slope (see Section G1114), so that $$D\quad = \quad \frac{P}{k}$$ where P is the load carried by the wheel in question. Unfortunately, however, there is a different problem. Furthermore, during the 1990s several luxury car models were produced in which a degree of rear-wheel steering was introduced to improve handling. In fact the windshield is associated with three distinct zones of separation: the trailing vortices from the A-pillars, the bound vortex (which remains in position rotating like a horizontal roller at the stagnation point at the foot of the windshield), and on some older cars, a line of separation at the leading edge of the roof. The clingfilm criterion How can you tell whether a force is 'external' or not? One might have expected the values to be higher in view of the need to stop the vehicle as quickly as possible. Hence, if one wheel passes over a bump or falls into a pothole, a car with a front beam axle will tend to veer to one side [14]. By contrast, on a bend, the centre of shear stress moves aft: lateral distortion of the tyre tread is zero at the front of the contact patch and grows to a maximum at the rear. Today, most front-wheel drive cars use a simple trailing arm or a variation on the trailing arm principle for the rear suspension, while rear-wheel drive cars are equipped with semi-trailing arms (figure 20), double wishbones, or occasionally MacPherson struts. Drag When a vehicle travels on a level road, the engine must produce enough power to overcome rolling resistance (see Section G0119) and aerodynamic drag. However, it is the first natural frequency f_{N1} that engineers are most concerned about, because it largely determines passenger comfort. It is controlled by a sensitive transducer inserted at some point within system to gauge the pressure exerted by the driver on the handwheel. This solution can be substituted back into equation 17, and by equating terms in $\sin (\Omega t)$ and then terms in $\cos (\Omega t)$, one can solve for C_1 and C_2 and finally show that the amplitude $A(\Omega)$ of the response, equal to $\sqrt {{C_1}^2 \; + \; {C_2}^2}$, is given by $$A(\Omega) \quad = \quad a \sqrt {\frac{\omega ^2 \left(\omega ^2 \; + \; 4 \zeta ^2 \Omega ^2 \right) }{ \left(\omega ^2 \; - \; \Omega ^2 \right)^2 \;\; + \;\; 4\zeta ^2 \omega ^2 \Omega ^2}}$$ What does all this amount to? equation 2 can be expressed in a slightly different way by substituting D for P/k: $$f_N \quad = \quad \; \left(\frac{1}{2\pi} \right)\sqrt {\frac{g}{D}} \quad \text{Hz}$$ As we saw earlier in the rubber band experiment in Section G1114, to minimise bouncing of the car body, the natural frequency should be as far as possible below the frequency of any waves or bumps in the surface. The mass of the quarter-body is M, the spring stiffness k, and the damping coefficient c. Will any springs do? Some practical implications Designers like to keep load transfer as small as possible, so that all four tyres are pressing equally on the road or nearly so, because the grip provided by a tyre does not increase or decrease in proportion with the normal contact force (see Section C1717); the gains and losses don't balance out. And if for some reason the driver lets go of the handwheel, it is vital that the steering centres itself automatically so that the vehicle continues in a straight line as opposed to veering off the road. Today, it is train manufacturers who are reviving the air brake as an auxiliary system for high-speed running. Again we can express the inner steering angle as a power series expansion of the outer steering angle and compare the coefficients with those required for pure rolling. They are all functions of δ_o and strictly speaking they should be written thus: $\sigma (\delta_o)$, $\delta_i (\delta_o)$, $\alpha (\delta_o)$, and so on. In fact most engineers believe that a fully 'active' suspension will be too expensive for mass-produced cars for many years to come. But unlike the equivalent test for a four-bar linkage, it doesn't offer any guidance about where to start. There are other elements in the suspension linkage such as rubber bushes at the hinges that provide additional compliance and affect the transmission of high frequency vibrations. There are differing views about the preferred direction of yaw. But they are not the only solution. The rubber band represents the suspension springs. No mention is made of the centrifugal force that is usually associated with objects moving on a curved path: the force that we feel tugging us outwards when we step onto a moving merry-go-round. In fact, comfort and road-holding are conflicting requirements. We know instinctively they must lean inwards, though we may not be able to explain exactly why. Both the double wishbone and MacPherson suspensions keep the wheel nearly upright and although there is some lateral track change, it isn't very much. Our aim in this section is to compare the demands made on the braking system by different types of vehicle ranging from bicycles to jumbo jets. With a reduction ratio of 16:1 to 18:1, the driver can turn the handwheel through 10-20° without necessarily losing control [4]. This is what one would expect. For most cars, the rear suspension is tuned to a slightly higher frequency. The resulting equation is rather unwieldy but it can be evaluated with $\sigma = \delta_{o} = \delta_{i} = 0$ to give $$\delta_{i}''(0) \quad = \quad \frac{2w}{s} \tan \theta$$ Appendix 3: The rack-and-pinion system Outer steering links Referring to figure 13, if we sum the lateral projections and then the longitudinal projections of the steering links OE and EF we get these two equations: $$a \sin (\theta - \delta_{o}) \;\; + \;\; b\cos (\phi + \psi) \;\; + \;\; h \;\; + \;\; \frac{1}{2} (s - w) \quad = \quad 0$$ $$a \cos (\theta - \delta_{o}) \;\; - \;\; b\sin (\phi + \psi) \;\; - \;\; u \quad = \quad 0$$ and differentiating equation 48 with respect to δ_{o} leads after a little rearrangement to $$\psi' \quad = \quad \frac{a \sin (\theta \, - \, \delta_{o})}{b \cos (\phi \, + \, \psi)}$$ which when evaluated at $\delta_{o} = 0$ becomes $$\psi'(0) \quad = \quad \frac{a \sin \theta}{b \cos \phi}$$ The next step is to get expressions for h, h' and h'' in terms of δ_{o}. Principle of rack-and-pinion steering Power steering Many drivers need some form of power to help them turn the handwheel on tight curves. It's easy to forget that when a horse or a vehicle travels round a curve, it is not in equilibrium and the forces acting on it don't balance. You can gauge it quite easily if you have a car handy and you are prepared to risk denting the bodywork. There is a six-fold reduction in the disturbances that get through to the passenger compartment. And let's treat the suspension spring together with the part of the vehicle it supports as an isolated system. The normal contact force at the rear is greatly reduced, and the maximum frictional resistance that it can contribute is reduced accordingly. The next step, if and when it happens, will be to de-couple the car body from the road surface altogether, so the vehicle floats along smoothly regardless of bumps and potholes. In simplified form, the relationship between p and V appears thus: $$p \; + \frac{1}{2} \rho V^{2} \quad = \quad \text{constant}$$ where ρ is the density of the air. Unsprung components must withstand heavier road shocks than sprung ones, and must be relatively robust. A lesser proportion comes from the wing mirrors and various other surface details such as door gaps and windscreen wipers. On a competition racing car, they glow red hot when the driver brakes on the approach to a bend. To understand what is going we can picture the air stream (a) moving smoothly in aggregate over the surface of the vehicle but (b) when viewed through a microscope, the molecules dart randomly in all directions within the aggregate. But engineers need to measure sensations so they can compare different vehicles, make selective improvements, and predict performance. First, the tyre of a cambered wheel behaves like a cone with its vertex at road surface level (figure 13). They arose at a certain critical speed and grew rapidly in magnitude until the driver slowed down, characteristics similar to those of 'hunting' in railway trains and 'flutter' in aircraft wings, of which more later. Load - deflection curve for suspension springing Nesting order A good way to reduce unsprung mass is to do away with rigid axles that stretch across the vehicle body, and replace them with independent suspension. Wind tunnel research has shown that the effects of quite small alterations, applied consistently to all parts of the vehicle body, can together produce a large drag reduction. One might describe this suspension set-up as 'hard'. Another way to reduce the risk is to make the handwheel resist motion when travelling at speed, and various manufacturers have experimented with hydraulic systems that stiffen the response accordingly. But it's not obvious how to achieve it using only simple mechanical components of the kind we find on mass-produced rail wagons and saloon cars. Stability Lift has a destabilising effect on a road-going vehicle. Any unintentional 'toe-out' in the plane of a rear wheel about its vertical axis will cause the vehicle to steer out of line. But it has lost energy through friction and may come to a halt before completing its journey. In a similar way, one can work out what happens if the damping is made more robust. Nowadays the unsprung mass associated with one wheel of a moderately large saloon is typically around 45 kg (100 pounds) [5]. If we denote by C the centripetal force required to hold the car in its curved path, and by w the track width (the lateral distance between the wheels), then the total increase in load δPcarried by the outside wheels is given by $$\delta P \quad = \quad Ch/w$$ There is a corresponding reduction in load on the inside wheels, and in an extreme case if the load falls to zero, the inside wheels will lift off the road surface and the car will roll over. One type of vortex forms when the body deflects the air so that its direction of motion changes from upstream to downstream. If you buy a supercar, you'll need a bigger garage. At very low frequencies, we expect the body to copy the wheel motion exactly. Ideally, the designer would shape the rear with an extended tail like that of the half-zeppelin, but if the tail were to enclose the passenger cabin and curve smoothly all the way down to floor level as shown in the upper part of figure 23, it would project an enormous distance behind the rear axle. Therefore, input and output are equal and the gain is 1.0. But as the frequency increases, the gain increases too, because when the wheel rises over a more localised undulation, the body 'bounces' on its springs. The six aerodynamic forces and moments acting on a vehicle Lift If you think of a car as a rounded hemisphere skating along the road surface like a beetle, it seems obvious that the air must rise like an arch over the roof and fall again in its wake. The zeppelin For aircraft, the teardrop or zeppelin (figure 2) is the shape that encounters the least resistance when moving through the atmosphere at any speed below the speed of sound. The damper (or shock absorber as it is sometimes known) is a device that, like the spring, is interposed between the wheel and the body. When the wheel descends downstream of the bump, the body is momentarily left behind. Lenkensperger's idea did not catch on in the UK, owing largely to conservative attitudes in the coachbuilding industry [1]. Like the stone in a sling, if there were nothing pulling it towards the centre of the ring it would fly off in a straight line. A trailing vortex is produced for example by an aircraft wing because it deflects the air through a small angle to produce a downwash. Additionally, a front-wheel drive car needs a constant velocity joint that connects the hub to the engine drive shaft. Except for heavy vehicles, irreversible systems are today less common than they used to be, but at one time, they included the screw-and-nut device, and later, the worm-and-gear. That is why the kingpin of a car suspension is angled inboard at the top: the axis around which the wheel swivels then passes close to the centre of the contact patch, which minimises the torque impulse from any sudden impact (see Section C1405). But during the last century manufacturers developed more efficient shapes, namely the torsion spring and the coil spring. POWLES, J K (1993) Implementing Line 1: Birmingham to Wolverhampton. As a result, the axle attachment on the outside spring moves rearward. It is an important part of any vehicle that rests on wheels, including railway trains and aircraft.The main purpose of the suspension is to smooth the ride for passengers (and cargo) over bumps in the running surface. Revised 19 February 2015 Page 7In order to steer a car round a curve, you have to swivel the front wheels so they point in the direction you want to go. The same applies to a mass sitting on top of the spring, or being pushed and pulled from side to side as shown in figure 11. The kingpin is arranged as close as possible to the central plane of the wheel. We now have four wheels all travelling along curves of differing radii, so we need to choose a reference point on the car that will represent the path of the vehicle as a whole. Toe-in To summarise, when you buy a car don't expect the front wheels to be upright; neither will they point exactly fore-and-aft even with the steering in its central position. For early cars, it was traditional to arrange the geometry so that the projections of the steering arms intersected at the rear axle (figure 8), which was thought to be a reasonable compromise [7]. For a rear-wheel drive car, a rearward load transfer during acceleration improves grip at the driven axle so the driver can make better use of the engine power without spinning the wheels. Tapering the rear Over the years, designers have tailored different models of car to the needs of different types of customer. Our aim in this Section is to show how the reality falls short of the ideal, and what manufacturers are doing about it. We shall refer to this imaginary configuration as the 'thin car model'. By contrast, the black curve represents the 'dynamic' behaviour with a rapidly oscillating input. The fixed geometrical parameters are shown in figure 12. Load transfer On a real road, variations in wheel load occur when the suspension rides over bumps and hollows in the carriageway surface. Fifty years ago, it was thought too difficult to design a family saloon that approaches the level of aerodynamic performance that is theoretically possible for an object of this size, and manufacturers gave a low priority to reducing air resistance. For example, in one particular system the upper steering column is joined to the lower via a spring that compresses when the handwheel is turned. WILSON, D G (2004, 3rd edn) Bicycling science MIT Press. MacPherson strut The MacPherson strut achieves a similar effect with fewer moving parts (figure 17). Air that doesn't enter the duct must flow smoothly up and over the engine compartment, avoiding any abrupt change of direction that might cause separation (figure 17). But there is another kind of energy to be taken into account. One must take into account what is happening to the wheel on the opposite side. The real vehicle can be pictured as two thin car models side by side spaced a distance w apart as shown in figure 5. While grip is thus maintained, no impact will be transmitted to the vehicle or, for that matter, to the road. In 1921, Austrian aeronautical engineer Edmund Rumpler used this as his template for the world's first systematically 'streamlined' vehicle. As a result, the flow pattern along the upper surface shows local separation in three areas, features that don't occur with 'pure' aerodynamic shapes. As shown in figure 1, the two forces combine to create a resultant force, and the horse leans until the resultant passes through its centre of mass. Here, we shall refer throughout to stiffness, load and deflection measured 'at the wheel' unless otherwise stated. The irregular profile of the underbody has a significant effect on drag, and some manufacturers now provide plastic fairings under the engine bay, and also recess the exhaust pipe into the floor pan [16]. Three modes of behaviour for a damped oscillating system Free oscillations Imagine the system freely oscillating after an initial disturbance (someone presses down on the bodywork and then lets go). The smoke trails indicate 'streamlines' in the pattern of flow as shown in figure 1, and they carry a message: this car slips through the air with ease. But the two halves of the steering column are also connected by a mechanical 'dog' that provides fail-safe reversion. But from a mechanical point of view, longitudinal movement is difficult to cater for and it would interfere with braking and road-holding, so engineers compromise by putting cushions in the suspension joints that absorb some of the shocks. The coach was then supposed to slither downhill with the wheel locked [2]. The aim is to minimise tyre scrub, so the front wheels roll freely and smoothly along their intended paths. Secondly, unless the side force acts at the neutral steer point (see Section C0418), it will also make the vehicle rotate or yaw. If a front wheel undergoes a sudden change of camber (leans away from the vertical) it tends to precess, twisting about a third axis mutually perpendicular to the axis of tilt and the plane of rotation. Nevertheless, the trend over the last few decades has towards smaller camber angles rather than larger ones. The system could then be treated as if it were in static equilibrium, with all the forces cancelling each other out. Unfortunately, the aerodynamic add-ons may not actually help. We could manage perfectly well without them. They will also deflect over bumps in the road surface. Revised 09 March 2015 Page 2Before the Industrial Revolution, travel was an uncomfortable business. In Europe, where cars are smaller and there is less room for the wheel to move up and down, passengers have always been accustomed to a bumpy ride. The wheel is assumed to move up and down as it traces out the fluctuations in z_{road}. Now for the experiment proper. In practice, of course, an automobile body responds simultaneously to roughness input through all four wheels, and the four suspension springs to some extent interact. When the car travels round a curve, the centripetal force tends to swivel the wheel further and increase the steering angle. The stiffness is the increment of load required to achieve unit deflection, and is equal to the slope of the curve when load is plotted against deflection as shown in figure 9. As mentioned earlier, drag forces on a car body have two distinct elements. It's not just the wheel that matters. Many different combinations have been tried, and the figures set out in table 1 illustrate the characteristics of different materials used on braking systems developed at different times during the last few decades (the coefficients of friction in the last row were culled from internet sites advertising motor racing and aircraft products). Can the zeppelin shape work for cars? When a wheel hits a bump, it rises. The yaw moment is therefore considered by designers as potentially more dangerous than the side force itself [19] [27]. Rubber springs are used on road vehicles of all kinds, and less often, on railway bogies. It was aerodynamically efficient, more so than most saloon cars of the 1980s [10], but it was not a commercial success. Fortunately, the problem can be treated by mounting the rear end of the spring higher than the front (figure 10). We cannot have zero scrub across the whole range. More important is the fact that almost half the surface area of a car lies within the wheel arches and underneath the body, all of which present rough cavities to the air stream. The movement can be repeated indefinitely. Let's call it f_{N1} for short. What the figures don't show is the effect of roll on corners. Independent rear suspension for cars was finally triggered by the widespread introduction of front-wheel drive, which removed the need for a continuous rear beam axle altogether. Since their role is to take energy out of the springs, it makes sense to dissipate the greater proportion of energy on the rebound stroke when the impact on the passenger compartment is minimal, and to provide relatively little resistance on jounce. NORRIS G and M WAGNER (2005) Airbus A380: Superjumbo of the 21st century St Paul: Zenith, see p98. Let's start by imagining a weight tied to the end of an elastic band (you may need to tie several small bands together, and a teacup will do for the weight but don't use the best china). Although the movements individually are quite small, their cumulative effects are not, and distortion under load affects the geometry of the system. The cooling duct was close to the ground where the air pressure at the front of the car was highest, an arrangement that allowed a small duct with minimum associated drag. The gearing ratio is commonly expressed as the number of degrees of handwheel rotation needed to produce one degree of steer angle. Let's assume it has linear properties and ignore the effect of the damper and the tyre (neither have much effect on N). Such a tail would also create lift together with trailing vortex drag [12]. How a beam axle induces camber change when passing over a bump The problem does not arise with independent front suspension, where the camber angle for a car tyre at full wheel jounce rarely exceeds 4% [20]. In recent hatchbacks, the cross section is reduced gradually downstream by tapering the roof, sides and under-body (figure 23). We shall also need to know the numerical value for this rate of change when the steering is in the central position, a quantity that we shall write as $\delta_{ip}'(0)$. With a conventional steering system on the rear axle, a vehicle that entered a corner too fast would quite likely spin round in a circle before the driver knew what was happening. But it was only an approximation, because for any given set of values of L, w, u and θ, it turns out that the 4-bar linkage is capable of producing 'pure' rolling only at isolated steering positions. The confused flow pattern around the suspension, floor pan and sub-frames raises the friction drag to over a quarter of the total drag for a modern car [7] [37]. Castor trail works fine on the front wheels of a vehicle because it is self-centring, a point that we have covered elsewhere (Section C1405). Suspension gain for a quarter-car model The ratio $A(\Omega)/a$ of the two amplitudes is sometimes called gain by analogy with amplification in electrical circuits (see, for example, [3]). On impact, the wheel rises at a speed controlled by the slope of the bump. Here, instead of the car body bouncing up and down on its springs, the wheel bounces up and down on the tyre. Let's begin by asking how the shape of the car has evolved through different phases in pursuit of aerodynamic efficiency. The result is shown by the blue line in figure 7. Otherwise any small deviation from the straight ahead position would cause the wheel to flip round 180 degrees (figure 11). But for ordinary cars, a high value lightens the steering task especially when parking, and a common figure is 16:1, although different specialists quote different ranges (see for example [4], [9] and [11]: also, cars with power-assisted steering manage with less). The cascade is hierarchical, with input from the primary suspension associated with each of 2 axles of a bogie feeding into the secondary system (figure 3). For any given wheel, the static deflection$\; D$ is the distance moved upwards by the wheel relative to the body when it settles [9]. Configuration for asymptotically pure rolling Errors with the configuration for asymptotically pure rolling The rack-and-pinion linkage The four-bar linkage was standard throughout the first hundred years of automobile construction, but has now been superseded by rack-and-pinion steering in which the rack slides from side-to-side in a guideway (figure 11). So the scope for camber is limited, especially on modern cars with wide tyres. Engineers describe the process as 'isolation' from surface roughness, which includes not only bumps and potholes but roughness on a smaller scale that produces high-frequency vibrations sometimes called 'harshness', together with audible noise. Briefly, this frequency needs to be as low as possible, for two reasons. However, it makes things difficult for the driver when the car is stationary. When its value is greater than one, the damper resists movement to such a degree that it prevents the body from ever reaching the equilibrium position. There is no prolonged bouncing up and down. In fact there are several natural frequencies, each associated with a different 'spring' in the cascade, but here we are interested in the one associated with the main suspension spring. Later, for the purpose of analysis we'll assume that they are directly proportional, so we can characterise the damper in terms of a single 'damping coefficient' c equal to the resistance force per unit velocity and measured in units of Ns/m, whose value is given by the slope of the graph shown in (figure 12). Rounding the front If you turn back to figure 1 you'll see that two of the streamlines terminate at the vehicle body, one at the front and one at the rear. Kingpin inside the wheel dish Positive camber Two-way roads are usually 'cambered', i.e, the cross-section is raised in the centre to promote drainage. On a sharp curve, the lateral distance between the front wheels was effectively reduced, which made the carriage more likely to overturn. The particles inside follow a helical path with the highest speeds occurring in the core. When compressed, oil must pass through a small orifice from one chamber to another, so the damper provides a resistance force that varies with the speed of deflection. Hydraulic damper Modern cars have dampers that are 'tuned' more precisely to the task, as do railway coaches. The distance between the centres of the two contact patches is called the track width. Notionally, we can cover it with clingfilm (figure 2). Mostly, this is done by tailoring the springs and dampers to suit the particular vehicle and the way we want it to behave. On the other hand, when the wheel encounters a downward slope and drops below its static position, the downward deflection is called is called rebound. Common sense suggests that a pointed nose would pierce the air more easily and produce less drag. The teacup will move up and down at the same speed as you hand, faithfully reproducing the input. It can be triggered by driving over cobblestones. Effect of varying track width Toe-out While lateral movements are a nuisance, swivelling movements are potentially dangerous. The degree of compensation can be gauged by measuring the actual steering angle $\delta_{o, actual}$ given to the outside front wheel and comparing it with the ideal value $\delta_{o,ideal}$ using the following formula (this is a simplified version of the one that appears in [2]): $$\text{Ackermann fraction} \quad = \quad \frac{\delta_{o, \, actual} \; - \; \delta}{\delta_{o, \, ideal }\; - \; \delta } \times 100\%$$ Typically, road cars have approximately 40% compensation at steering centre, possibly rising to 60% at full lock [3]. Notice how the dampers smother the motion so it dies away in less than two cycles. Attached to each kingpin is a steering arm, and the two steering arms are connected by a track rod running across the width of the front suspension. You can see the impact – and hear it – if you stand on the footway next to a busy road and wait for a pick-up truck to pass over a nearby speed hump or pothole. This is the condition required for pure rolling. Conversely, if the boundary layer remains attached throughout, the pressure drag is theoretically zero. In the case of a horse and rider of mass M travelling at constant speed V round a curve of radius R, the lateral acceleration is V^{2}/R acting towards the centre of the curve. At this point the boundary layer reverses direction or separates, leaving a turbulent wake (figure 14). The names are not very informative but there is an important difference between the two systems. The vibration is a mixture of frequencies representing the irregularities that make up the road surface profile, all of which feed into the wheels simultaneously. The first part is carried out very slowly. These wheel motions form the input. The criteria that a braking system must satisfy vary considerably according to the conditions under which the vehicle operates, and the forces that passengers can tolerate. There seems to be broad agreement that the lateral centre of pressure should be aft of the centre of mass or at least close to it (see, for example [20] [26], and also the extensive discussion in [32]). We'll be looking at some examples for road vehicles shortly. It senses the driver input and augments it via a ram that engages with the drop arm or track rod. Parallel Ackermann steering Ackermann geometry At this point we pause for a moment, and imagine the car as having only a single wheel at the front and a single wheel at the rear. So the relationship between the two steering angles matters, and until recently the mechanism that was used to control this relationship was one of the simplest devices that you could imagine: three rigid bars connected together at two pivots (figure 1). This is why car designers try to keep down the weight of the suspension, with coil springs rather than leaf springs, for example, and light alloy wheels on the more expensive models. Longitudinal transfer Suppose the wheelbase of the car is L and its centre of mass lies a distance h above the road surface (figure 4). Both arise from the shape of the vehicle body rather than its surface area as such. The 'Hotchkiss' layout with longitudinal prop shaft, beam axle and rear leaf springs persisted for many years, and variants of this layout are still used on light vans and trucks (figure 19). To find out, we first need to establish the theoretically 'perfect' shape. At high speeds, the steering angle is barely noticeable, because quite a small change can fling the car sharply to one side. It is called the d'Alembert force. We'll distinguish the pure rolling inner steering angle by using the notation δ_{ip}. It confirms what we might suspect, that to a first approximation the steer angle of the inside front wheel changes at the same rate as the steer angle of the outside front wheel. The angle of the rear window or 'backlight' is critical, and it became a significant issue, for example, in the development of the Volkswagen Golf I [39]. The reduction ratio can be lowered at the same time, giving a more responsive steering feel. There is, however, a drawback: the greater the rake, the greater the mechanical trail, and the greater the self-aligning torque. But the two points don't coincide, and neither point lies at the centre of the contact patch. When the steering is in the dead-ahead position, each lever arm makes an angle θ with the car's longitudinal axis, and the track rod is spaced a distance u rearward of the front axle centreline. The teacup represents the car body, and its motion constitutes the output. But you will have noticed from figure 9 that the front wheel housings and the driver's cabin break up the outline so that no part of the vehicle body actually presents a totally wedge-shaped profile to the oncoming airstream. Steering feedback through castor trail Steering systems The most widely used steering system today is rack-and-pinion steering, which was pioneered by BMW in the 1930's [17]. The point we choose is the centre C of the rear axle. You can download the paper by clicking the button above. For example, Citroen gave its 1970 Citroen SM its power steering with artificial 'feel', in which the steering grew heavier at speed [18]. Stopping distances During the eighteenth century, a horse-drawn carriage was usually equipped with a drag shoe, a portable device like a shovel that could be wedged under the rear wheel to prevent it rotating. This roughness comes from individual stone particles embedded in the road surface, typically on a scale of just a few millimetres across. The parameters of the signalling framework are fixed so that if the train is required to stop for a signal, the rate of deceleration is around $0.05g$, although higher rates of deceleration are possible in an emergency. In this section we shall try to work it using a rigorous mathematical criterion. They are: $$\delta _{ip}'(0) \quad = \quad 1$$ $${\delta _i}''(0) \quad = \quad \frac{2w}{L}$$ Substituting the results in equation 11 gives $$\delta _{ip} \quad = \quad \delta_{o} \;\; + \;\; \left(\frac{w}{L} \right)\delta _{o}^2 \;\; + \;\; \text{terms in powers of } \delta_{o}^3 \text{ and above}$$ Now for the four-bar linkage. Out-of-phase displacements of the wheel and body Industrial products are often tested in the laboratory to see how they respond to loading. A sudden gust can blow a vehicle off-course, and occasionally even turn a goods vehicle onto its side [24]. It increases rapidly with V, and at higher speeds the load carried by the rear axle of a hatchback may fall by as much as 10% [25]. Internal forces include the car's weight together with the so-called inertia forces of which the centrifugal force is one example. The teacup will bob up and down at a steady rate, the natural frequency of the system. Put $\Omega = 2 \pi V/L$. During the 1960s, the body contours changed: the family saloon assumed a 'three box' configuration with the wheels fully enclosed (figure 6). And since the engine compartment is usually overcrowded, for front-wheel drive cars there may not be room for the track rod immediately behind the kingpins [8], and the tracking assembly is sometimes mounted ahead of the axle. How springy do they have to be? But if anything, the physical effort needed to steer at low speeds has increased with the trend towards heavier cars together with front-wheel drive and wide tyres. When it goes round a curve, the front and rear wheels follow paths of different radius, so we must choose one of them to represent the motion of the pair. Close to the natural frequency, the gain nearly doubles. If the angles taken up by the two front wheels are not compatible, they will 'scrub' the road surface and slow the vehicle down. 'Idealised' means that the spring is linear and neither the mass nor the spring are affected by friction or damping. The extra length is taken up by the shackle through which the trailing end of the spring is attached to the chassis. Above all, the driver wants maximum grip, and soft springs don't keep the wheels in continuous contact with a bumpy road surface at the higher natural frequency where the unsprung mass is prone to resonate. Clearly, we must increase the angle θ a bit more yet, but to what value? There are two other reasons for reducing unsprung mass: A large unsprung mass increases the severity of impact between the wheels and the road, which in the case of heavy lorries, can damage the road surface. The heavier the wheel the greater the momentum, and the greater the impact on passengers and cargo. They include as before the wheelbase L, track width w, track rod set-back u, steering lever arm length a, and steering arm angle θ. The central 'rack' has raised teeth, and it is enclosed in a steel tube. It can be used to explain why a double-decker bus will tip over if driven too fast round a bend. Hydraulic servo assistance provides 80 - 85% of the power needed for heavy vehicle steering, and 70 - 75% of the power needed for saloon cars [12]. In this critical regime, although the lateral force grows throughout, the pneumatic trail decreases and the two compensate, so that the self-aligning torque reaches a plateau [6]. The centripetal force that keeps the horse's hooves inside the ring is pushing as it were against an open door. Occasionally, large impacts will carry the suspension towards the limit of its travel. But the drivers of those days saw little virtue in brakes, and rarely used them. They are activated by electromagnets whose magnetic field attracts them towards the rails. For the time being we'll assume the wind is coming from straight ahead. But in the case of braking, there is an additional complication. All round, therefore, a hard suspension is better for handling. The wedge Aerodynamics Air flow over a moving vehicle affects its behaviour and performance in several ways. And yet, if we encounter an icy patch on a bend, we expect feedback – we want the handwheel to provide 'feel' of the road surface conditions at all times. In this way, the designer can raise the angle to more than 10$^\circ{}$ [10]. Perhaps as a result of growing concern about energy consumption and greenhouse gas emissions, manufacturers have revised the profile in subtle ways to reduce air resistance without compromising other requirements. Each impact has firstly a component parallel to the skin, and the cumulative effect is a shear stress whose value varies in magnitude and direction everywhere around the envelope (figure 10). Since a cone rolls naturally around its vertex, a wheel that leans to the right will steer to the right, and one that leans to the left will steer to the left. Analysts usually divide the de-stabilising effect of a crosswind into two distinct components. It follows that precise engineering of the steering and suspension is needed to ensure that the driver's hand movements are communicated faithfully to the front wheels. When applied, they clamp themselves to the rails and bring the tram to a halt. By contrast, with positive trail the car is directionally stable even if the driver releases the handwheel. Here, we observe the effects of high-frequency input (figure 3). Briefly, the springs (a) are ineffective at low frequencies, but they don't do any harm, (b) work well at high frequencies, by intercepting surface roughness and isolating the vehicle, and (c) cause trouble at the natural frequency of the system. Worse, the running surface could be wavy, and the waves in the surface could deliver regular pulses at the natural frequency of the suspension and make it resonate. We think of the air stream as a collection of tubes arranged parallel to one another in the direction of flow. When a leaf spring deflects, it flattens and therefore its horizontal length increases as shown in figure 8. A characteristic feature of this motion is that the wheels and body don't move in the same direction at the same time. Payload When fully loaded, a saloon car will carry a payload of passengers and luggage equivalent to about 40% of its unladen weight. But our experiment should convince us that springs alone are not enough. The position of the stagnation point S1 determines what proportion of the air flows underneath [13] [30]. This is particularly important for the wheel on the outside of the curve, which carries a greater load and is responsible for the greater part of the cornering force necessary to keep the car on the road. As the angle increases further, the drag falls again, and in fact a squareback estate can have lower drag than a notchback based on the same body shape [14]. The load is then removed, and provided the relaxation is gradual, the load and deformation will trace out the same curve in reverse. This is further forward than the traditional layout for early cars, and as the technology progressed some manufacturers did indeed move the intersection point forward to a location roughly two thirds of the wheelbase from the front axle [2]. The zeppelin has a low drag coefficient, but the long tail presents a problem and the cylindrical cross-section is not very convenient. When a front-wheel drive car emerges from the side road, you will often hear the tyres slip and shudder as the driver tries to accelerate quickly into a gap in the traffic. With a high castor angle, when the driver enters a curve the front wheel is presented to the road not vertically but leaning inwards like the front wheel of a motorcycle, which reduces the slip angle and makes the steering more precise. Input at natural frequency Now the final part. Roll-induced camber with independent suspension Camber steer Suspension linkages As mentioned earlier, we want each wheel of a car to be capable of up-and-down movement without any unwanted deviations from its static position. On a sharp curve, the body rolls, causing increased deflection on the 'outside' spring and reduced deflection on the 'inside' spring. Briefly, the car should be long, low and wide, with a wheel at each corner (figure 7). The grey line represents the load-deflection curve under gradual or static loading, and is similar to the one shown in figure 9. Since there is mixing between the surface layer and neighbouring layers, the friction 'drag' permeates some distance into the surrounding air stream. At the start of this range, around 10 Hz, lies a second natural frequency f_{N2} called wheel hop [5]. On the whole, vortex drag is less important than the other types. It arises because the boundary layer doesn't always stick to the body over which it is flowing [8]. It doesn't. One thing is certain, that it was not Mr Ackermann. The tyre is attached to the wheel, which itself is slightly springy and absorbs a certain amount of shock loading, as do the elastomer joints mentioned earlier. It follows that the system as a whole behaves like a series of springs and dampers arranged in the form of a cascade. The sum of these forces is $Mg - k(D + z) - c\frac{dz}{dt}$. Let us imagine a half-zeppelin skimming along the road with an infinitesimally small gap between the underside of the vehicle and the road surface (figure 21). By contrast with the jounce stroke, unless the car is travelling quite slowly, the steepness of the slope is irrelevant. Angle between engine cover and windshield The rear But the aerodynamic efficiency of a saloon car is not very sensitive to its shape at the front. Pneumatic trail Feedback from self-aligning torque As with mechanical trail, pneumatic trail exerts a torque on the steering that tends to make it return to the central position. Without altering the pattern of airflow or the drag coefficient we can in our imagination replace the semi-zeppelin with a whole zeppelin whose lower half takes on a virtual existence below the road surface together with its virtual streamlines. Similarly, to maximise cornering grip, one makes h as small as possible and w as large as possible. Unsprung mass There is another difficulty. We might want, in other words, the actual curve to converge asymptotically on the ideal curve as the steering angle δ_o approaches zero. As already noted, with an independent suspension, if the whole car leans, the wheels lean with it, and this tends to lift the inside edge of the tyre clear of the road surface. A little later came the D-type Jaguar competition sports car. In equation 12, we can immediately put $\delta_{i}(0)$ = 0. The rate of deflection of the wheel in jounce is determined by the slope of the bump in the road surface, in the sense that a steep ridge will ram the wheel sharply upwards, and a shallow ridge less so. For heavy trucks, where the payload can amount to several times the weight of the vehicle itself, the problem calls for a more radical approach, for example using air springs whose stiffness can be varied automatically to suit the loading conditions. But when we look at individual wheels, the situation changes. NUNNEY, M J (1998) Light and heavy vehicle technology Butterworth-Heinemann, see p561. Typically, the value of C_{L} for a passenger car lies between 0.2 and 0.5 [47]. What do we expect to find? You'll have noticed that figure 1 only shows the forces acting on the horse's feet. It would be nice if it allowed the wheel to move backwards and forwards as well (figure 5), because the impacts delivered by a rough road surface are not just vertical - they have a longitudinal component too. Does the same principle apply to all modes of transport? But if you look carefully, you will notice there is something odd about the geometry when the driver steers round a sharp curve and the wheels are at full lock (figure 1). We want the error to be small especially for low steering angles because drivers spend most of the time with the handwheel close to the dead ahead position when driving at motorway speeds. This gave some relief from the jolts induced by potholes in the road surface, but it produced an unpleasant swaying motion that made passengers feel ill. Whenever the car is influenced by an external force, the film will reveal its presence, showing up for example any aerodynamic disturbance together with contact forces between the tyres and the road. The rider is not pulled over the handlebars: it is the wheels that are dragged back underneath the rider and not the other way round. Not only will it be lighter, it will fit more easily into a cramped engine compartment, and allow the designer to program any chosen regime of steering characteristics into the system that might be thought desirable. Volume 3: 1960 - present day London: Guild Publishing, see p174-5. A rounded shape introduces special difficulties, because the point at which the airflow separates (around the rearmost roof pillar) can move, leading to a sudden increase in yaw moment as the intensity of the crosswind increases. With your free hand, gently pull the teacup down a couple a centimetres, then let go. Porsche 356 with all-enclosing one-piece body shell The result was a graceful design that turned out not to be as efficient as one might expect. There is another requirement for good handling. This conclusion remains substantially unchanged even if we increase or decrease the distance u by an appreciable amount. The clingfilm criterion When describing the circus horse earlier, we ignored what was going on inside the skin of the horse and the skin of the rider and instead looked at the arrangement of external forces in relation to the horse's centre of mass. Hence the pressure drag is roughly proportional to the cross-sectional area of vehicle at the point of separation of the wake. This is the approach followed in almost all systems in use today. Only much later in 1878 did Charles Jeantaud succeed with an improved layout in which the wheels turned through different angles [2] [14]. Momentarily, the suspension is stiffer than it would otherwise have been, so its natural frequency rises and it becomes less effective at absorbing low frequency impacts that would otherwise have been the case. Wind yaw angle Yawing away from a crosswind So what can the designer do to ensure that the car is stable? Details can be found in many excellent publications (see for example [3] [6] [9] [11] [13] [18]). Steering angles for pure rolling Traditional Ackermann set-up But in reality there is no value of θ for which the linkage shown in figure 6 reproduces these steering angles exactly over the whole range of handwheel movement from left lock to right lock. Since then, the situation has changed completely, because the modern trend is to go the other way [1]. In other words, if you are designing a powerful car, you will make sure that the brakes match the performance of the engine so they can bring the vehicle to a halt quickly enough to give protection to the occupants and other road users as well. There may also be a net lateral force if the shape of the body is not symmetrical or if the air flow is not parallel to its longitudinal axis, and there are potentially three aerodynamic moments that tend to make the vehicle rotate (figure 13). However, in the case of a motor car, the principles can be illustrated with a two degrees-of-freedom model with the tyre at the first tier and the spring-and-damper at the second, or even a single degree-of-freedom model with spring and damper only. Components of the system The starting

point for an effective suspension is to mount the body on springs. Schematic anti-roll bar This why cars with independent front suspension almost always have an anti-roll bar, whose purpose is to limit the roll to an acceptable level (figure 18). Unfortunately, when the driver reduces power, for example when braking or coasting downhill, this no longer applies. Therefore, precise management of the suspension geometry is vital. Sliding begins at the rear of the contact patch and moves progressively forward as the demand for grip rises. There are countless combinations of the six values, but if one chooses a combination that happens to satisfy equation 24, the steering system will produce zero tyre scrub at small steering angles. To design an effective braking system you need to know how it will be used, and in particular, the maximum energy flows it must handle. It is not totally rigid, but flexes in torsion to allow some independent movement of the two front wheels. Vortices created by the front of a passenger car Vortices and wake at the rear of a passenger car The ideal shape So what is the potential for improvement? Another good aerodynamic shape The four-door saloon Revolutionary designs have rarely appealed to the motoring public, whose tastes are largely conservative, and it was many years before aerodynamically efficient bodywork became acceptable. On passenger cars today, the track rod has been superseded by a ‘rack-and-pinion’ system, but we’ll ignore this for the time being because it doesn’t greatly affect the geometry. They are transmitted through the suspension to the vehicle body whose motion constitutes the output, with different frequencies attenuated to a greater or lesser degree by the spring and damper combination. So if the same braking torque is applied to the front and rear wheels, the lightly loaded rear wheels will lock up and as a result, lose some of their braking force. The cascade At the lowest level, the tyre acts as a stiff ‘spring’, absorbing high-frequency vibrations (between 20 and 400 Hz) that are associated with small-scale surface roughness. The dimensionless quantity ζ symbolised by the Greek letter ‘zeta’ is known as the damping parameter. By repeatedly pressing down at intervals you will find the value of f_{N1} is probably a little over one cycle per second (1 Hz). Therefore we’ll take L, u and w as given. Now for the pressure drag and vortex-induced drag. One way to quantify the action of a suspension is to think in terms of input and output. As it does so, the track rod shifts to the left and rotates anti-clockwise through an angle σ. This is the principle behind the emergency brakes used on modern trams. Lift doesn’t actually slow the vehicle down, although it can affect the handling as we’ll see later. Finally, there must be enough room inside the wheel dish for the whole assembly to flex as the springs jounce and rebound over bumps in the road, and there must be enough room for the wheel to swivel, without any of the components fouling one another or fouling the bodywork. Although fashionable for sports cars, the main advantage in comparison with metal disks is the weight saving, and the ability to operate at high temperatures. German coachbuilder Georg Lenkensperger built a prototype in 1816 in which the wheels swivelled independently but remained parallel (figure 3). This relationship shows how the load on the wheel affects the ‘softness’ of the ride: if P doubles, the natural frequency decreases by a factor of $\sqrt 2$, and therefore the heavier the load, the slower the bounce. So however powerful the brakes themselves might be, the braking force is limited by the coefficient of friction between the tyre and the road. Effect on errors of increasing the lever arm angle to 21.5 degrees However, we are looking for a set-up such that the error is close to zero for a wider range of values of δ_o, in other words the curve lies close to the x-axis in figure 7 not just in two places but over a considerable range. But in practice, it’s the shape at the rear that matters, because the shape at the rear determines how and where the boundary layer separates from the vehicle body and hence the cross-sectional area of the wake. However, electrically powered systems are now preferred to hydraulic ones, because they can be managed with digital control routines that simultaneously perform other functions such as traction control. Revised 12 March 2015 Page 5The front wheels of a car spend most of their lives hidden under the wheel arches. Air ‘leaks’ around each wingtip from the underside to the upper surface, and the helical motion becomes a trailing vortex as shown in figure 15. How the tyre tread ‘averages out’ surface roughness All these processes help to minimise the amount of energy that is passed up the chain before reaching the vehicle occupants, and they all work in more-or-less the same way. You can observe the effect if you stand near to a T-junction in wet weather. Drivers can use this downforce to tackle curves at higher speeds than they would otherwise do, which is how races are won or lost. Spring force: since the spring deflection is $D + z$, the spring force (measured at the wheel) is equal to stiffness times deflection or $- k(D + z)$. When travelling on a curve, the body leans outwards, and as figure 12 shows, the wheels lean with it. Not only does suspension make the journey more comfortable, but it improves wheel grip and reduces shock loads so the vehicle body is less likely to suffer through progressive distortion or fatigue cracks. Better still, if the springs are designed with less curvature, the system can be made to understeer [19]. On a circular track, if C moves along a curve of fixed radius R, then as shown in figure 5 the inner pair of wheels follow a curve of radius $R - (w/2)$ and the outer pair a curve of radius $R + (w/2)$. The equations are simpler if we choose the rear wheel. But on closer inspection we see that the slope is not constant: the inner wheel must turn through progressively larger increments relative to the outer wheel as the driver turns the handwheel to the right. Long after springs were introduced during the 17th century, the leather straps remained, and today we still refer to any mechanical system intended to soften the ride as a suspension. Only one of the remaining three parameters can be varied at will – whichever one we choose, as soon as we fix its value the shape of the mechanism is fully determined. So on a real car, the suspension will deflect away from the reference position from time to time. These are the conditions under which suspension works best. When the car is moving, the suspension meanders up and down relative to this static position. The piston-and-cylinder on the right-hand side represents a damper. This may be compensated by a small toe-out. The size and ducting of the inlet to the radiator together with flow inside the engine compartment has an appreciable effect on drag, contributing as much as 10-15% of the total [5]. During any journey, the car will roll a little from side to side on curves, which causes the wheels to move up and down relative to the chassis. The wheels raise the drag coefficient to about 0.15 [44]. Towards the rear, as the air curves around the sides and begins to converge, it slows down and the pressure rises as one might expect. On the other hand, a light wheel passes on a less energetic impulse, and in particular, reduces the intensity of ‘wheel hop’ at around 10 Hz. And it stays in better contact with the road surface and therefore improves roadholding. At today’s speeds, the tyres of a standard road vehicle do not actually roll in the direction they are pointing, and the ‘slip angle’ adds a further dimension that takes the problem beyond our scope. But this is not a typical position. The coil springs can be placed further apart, effectively at full track width as opposed to not much more than half the track width typical of leaf springs. The spring controls a valve that in turn determines the degree of assistance. It will elongate roughly in proportion to the applied load, with the values of load and deformation tracing out a curve such as the grey line shown in figure 9, where tension is taken as positive so the trajectory starts off towards the right-hand side of the graph. That leaves a and s, whose values can be calculated from the others thus: $$\begin{equation} a \quad = \quad u \sec \theta \end{equation}$$ $$\begin{equation} s \quad = \quad w \; - \; 2a \sin \theta \quad = \quad w \; - \; 2u \tan \theta \end{equation}$$ Now for the variables. At some intermediate value the teacup will start to oscillate violently (figure 4). But there were notable exceptions. It is also lighter, less cumbersome to manoeuvre, and less prone to damage. Rubber springs Compressed air spring Spring characteristics Most steel springs are elastic, that is to say, when they are squeezed or stretched, the deflection is proportional to the applied force, and vice versa, at least for loads within a certain range. The low-frequency bumps and waves will become much more noticeable for passengers. The gradual taper discourages the boundary layer from breaking away until the last possible moment, and by reducing the cross-section of any wake, minimises drag. You can see from the diagram that the steering angle of the outside front wheel must be smaller than δ because its path intersects the vehicle chassis at a shallower angle. They work by reversing the mode of operation of the motor, so that it becomes a generator that is powered by the wheels. But they were unstable because it was necessary for each rear wheel to trail slightly behind its kingpin axis like the castor on a supermarket trolley. In combination with castor angle, this explains why the front wheels of a parked car sometimes appear at an odd angle. Their overall effect can be pictured as having two main components: drag, which acts against the direction of motion, and lift, which acts upwards, at right-angles to the direction of motion. Few people ventured far from their homes, and only the well-to-do travelled by coach. Anything more complicated would have been difficult to make, but the single pivot wasn’t ideal. At one level, it determines how dirt splashes around the sides of the body and across the rear window, and it determines the amount of wind noise, which can be distracting for passengers when travelling at motorway speeds. The tyres work mainly by wrapping the tread around the projections so that the road surface texture is effectively ‘averaged out’ (figure 3). Friction between the leaves as they bent up and down tended to absorb vibrational energy quite quickly. Friction There are three kinds of brake in common use: the friction brake, the electromagnetic brake, and the aerodynamic brake, each of which dissipates energy in a different way. A mother will stop pushing if the child seems to be at risk, but in a mechanical system, if there is nothing to check the accumulation of energy, oscillations can build up until something breaks. Let’s deal with the pressure drag first. Early carriages were designed with the front axle mounted on a single pivot on the vehicle centreline (figure 1). Appendix 1: Steering angles for pure rolling figure 8 shows the car with the outer and inner wheels swivelled respectively through steering angles δ_{o} and δ_{ip}. On each wheel of a car, for example, a pad presses against the metal surface of a disk or drum. Imagine you are holding in your hand a motor car, albeit the car is hanging upside down. The pressure in the wake is less than it would have been had the boundary layer remained attached, and like a great suction pad, tends to slow the vehicle down. At first, the rise is taken up by compression in the spring, but the spring now contains stored energy and this energy must be passed on to the vehicle body albeit at a reduced rate. Finally, we plot the error curve for this steering geometry applied to the car that we looked at earlier. In principle, the effects can be analysed using computer simulation coupled with a statistical representation of the road surface, whose general form is sketched out in section C1603. In general terms, the car is less responsive to steering manoeuvres and the driver doesn’t have a good ‘feel’ for what is happening to the tyres. For minimum drag, a zeppelin must be flying well clear of the ground so the air can flow freely underneath as well as round the sides and over the top. It does this because any steering movement will generate a cornering force on the tyre contact patch that in turn restores the steering towards its central position (figure 3). We have to imagine the nose pierced with a circular orifice to supply cooling air to the engine (figure 8). On a racing machine, a long tail adds weight, increases aerodynamic lift over the rear axle, and is prone to damage, so the potential saving in drag isn’t worthwhile. Nowadays it looks rather quaint, and in fact it interferes with good road-holding because positive camber reduces cornering stiffness. Conclusion If the purpose of the Ackermann steering linkage is to minimise scrub at small steering angles, the theoretically optimum layout requires the point of intersection of the lever arms to be located close to the centre of the vehicle wheelbase. The normal pressure can be pictured as acting at a point. Performance at the higher frequencies is improved, but at a price. Pivot axis inclination When viewed from the front of the vehicle, the pivot axis of a modern steering mechanism leans inward at an angle of up to 15° [3], while the camber value is 1.5° or less [4] [7]. The stabilising effect of negative offset Conclusion As if all this were not enough, when the front wheels of a car are set in the ‘straight ahead’ position, the wheels don’t actually point fore and aft (figure 15). The way to do it is not to make the car look like an aircraft. The idea is to press down vigorously on one of the front wings (fenders) and immediately let go. So the horse re-arranges its geometry, using the upward thrust N on its hooves to balance the centripetal thrust C so it doesn’t fall over. Additionally, within the front suspension there must be a hinge around which the wheel pivots to provide steering. The name ‘teardrop’ is perhaps misleading because in free fall, a drop of water will flatten into a disk at right angles to the air flow. Assuming a constant rate of deceleration, one can easily estimate the corresponding energy dissipation rates, which are shown in the last column. Here, we’ll concentrate on stability. Inside the wake, the air pressure is lower than the pressure acting on the front of the car, and the resulting pressure difference is responsible for a major proportion of the overall drag. As the zeppelin descends towards the ground, the flow of air underneath is partly obstructed, and most of the streamlines curve upwards and arch over the top. Sports cars don’t necessarily perform any better than the mass-produced hatchback. Most cars lean or roll outwards on corners so that the passengers slide in their seats towards the outside of the curve. Therefore its rate of acceleration is independent of the shape of the depression over which it is travelling and independent of the vehicle’s speed [4]. In figure 3, the cosine rule can be applied to the angle at vertex O in triangle OIM to get $$\begin{equation} e^{2} \quad = \quad a^{2} \; + \; w^{2} \;\; - \; 2aw \cos \, (\delta_{o} + \frac{\pi}{2} - \theta) \end{equation}$$ Applying the cosine rule again, this time to the angle at vertex I in the same triangle gives $$\begin{equation} \cos \alpha \quad = \quad \frac{e^{2} \; + \; w^{2} \; - \; a^{2}}{2ew} \end{equation}$$ And again to the angle at vertex I in triangle MIN: $$\begin{equation} \cos \beta \quad = \quad \frac{e^{2} \; + \; a^{2} \; - \; s^{2}}{2ea} \end{equation}$$ Finally, since the angle OIQ is a right-angle, we know that $$\begin{equation} \alpha \; + \; \beta \; + \; \delta_{i} \; + \; \theta \quad = \quad \frac{\pi}{2} \end{equation}$$ or $$\begin{equation} \delta_{i} \quad = \quad \frac{\pi}{2} \; - \; \alpha \; - \; \beta \; - \; \theta \end{equation}$$ In principle, this equation quantifies the relationship between the inner and outer steering angles for the four-bar linkage. If the upper wishbone is made shorter than the lower one, the outward lean of the wheel is reduced on corners, while at the same time, lateral scrub is minimised. The convention is to regard all the forces as acting at road surface level halfway between the front and rear axle on the vehicle centreline. This adds weight, which both increases the fuel consumption and increases manufacturing costs. Here we shall concentrate on a few examples, using them to illustrate some practical aspects of suspension geometry. First, let’s see what happens when the offset is positive. The resulting cornering force together with the unbalanced braking force on the right-hand front wheel tends to rotate the car bodily so that it veers to the right (figure 13). Even at a modest cruising speed of 80 km/h (50 mph), provided the car is travelling on a straight, level road, aerodynamic forces swamp all the other influences such as rolling resistance. You can confirm this by leaning on a car and letting go. If there are waves in the road surface, the wheel moves up and down with the waves. A typical value for a passenger car is about 0.3, so when cruising at motorway speed, say 120 km/h or 33.3 m/s2, given an atmospheric density of 1.226 kg/m3, the drag force comes to $$\begin{equation} 0.5 \times 1.226 \times 33.3^2 \times 0.3\quad \times 2.0 = \quad 408 \; \text{newtons} \end{equation}$$ Because the drag is proportional to V^2, a 10% increase in V will lead to a more than 20% increase in drag and hence an appreciable increase in emissions and fuel consumption. It is therefore necessary to have supplementary springs that progressively stiffen the response at either end of the range. Of course, moving components up the cascade is equivalent to moving the springs down. Rather, it is pushed downwards by the spring at a rate determined by the stiffness of the spring moderated by the damper. This is why the dampers of a car suspension are tuned to respond differently to jounce and rebound. Usually for a rear-wheel drive model, they are adjusted with a small toe-in (typically less than one degree). They can occur with either front or rear suspension but for simplicity we’ll concentrate on the rear wheels. Acknowledgement Photo on opening page: Jaguar front suspension by Joanna Walpole. The situation for aircraft is more complicated. Among the alternative forms of rear-end treatment are the traditional ‘notchback’, the hatchback, and the fastback (figure 24). The values of the coefficients $\delta_{i}'(0)$ and $\delta_{i}''(0)$ are derived in Appendix 3, equation 65 and equation 66. The spring provides most but not all of the flexibility: all the component parts deform under load to a certain degree, especially the pneumatic tyre. With certain types of suspension, continual jounce and rebound of the suspension while the car is in motion will quickly wear out the tyres, and on a curve, the lateral forces may interfere with directional stability. Evolution of streamlining 1920 - 1940 The three-box configuration Sports cars Of course there were sports cars too, low and sleek like the 1950s Porsche series of rear-engined machines. How does the spring-damper combination respond when it is squeezed and extended? Natural frequency Another important characteristic of a suspension system for any given wheel is the natural frequency$\; f_N$, the frequency at which it will oscillate after receiving a single jolt. Railways have been doing this for a long time, and their efficiency continues to improve with more sophisticated forms of moving block signals, but for motorists, a cooperative system is some way off. Ideally, the pivot axis should pass through the centre of the contact patch. The d’Alembert principle applied to the circus horse For the time being, however, we will continue to relate the external forces to the centre of mass. Inclination of the pivot axis on a modern double wishbone suspension Swivelling the front wheels jacks up the car More on offset It is now time to return to the question of why we need camber or pivot axis inclination at all. BOWDEN, F P and TABOR, D (1974, 2nd ed) Friction: an introduction to tribology London: Heinemann, see p143. Put crudely, less air underneath means less drag, so the first priority is to locate S1 together with the cooling inlet duct as low down as possible. As will be explained in a moment, for a saloon car an ideal value for the static deflection is about 250 mm (10 inches), but most cars have less, with small cars around half that much [4]. But priorities have since changed. First, consider friction drag. We’ll say more about this configuration this later. But superimposed on these reactions are loads that arise as a consequence of braking, acceleration and cornering. Built in 2005 by the East Japan Railway Company, the prototype tilting Japanese bullet trains ‘Streamline’ and ‘Arrowline’ are designed to run at 360 km/h (225 mph). Acknowledgement Photo on opening page of Mini cornering at Goodwood by Joanna Walpole. The inner wheel is not swivelling far enough, and the shortfall rises almost to 10% near full lock. The effect can be noticeable even at low speeds in parking manoeuvres. Abolishing inertia? A more radical approach involves re-positioning suspension components higher up the cascade; for example, one can mount the brake discs inboard of the springs. They are: $$\begin{equation} \delta _{i}'(0) \quad = \quad 1 \end{equation}$$ $$\begin{equation} \delta_{i}''(0) \quad = \quad 2\tan (\theta \, - \, \phi) \left[1 \;\; + \;\; \frac{a}{b} \sin^{2} \theta \sec^{2} \phi \, \text{cosec}(\theta \, - \, \phi) \right] \end{equation}$$ If we substitute these expressions into the Maclaurin expansion equation 12 we get: $$\begin{multline} \delta_{i} \quad = \quad \delta_{o} \;\; + \;\;\ tan (\theta \, - \, \phi) \left[1 \; + \; \frac{a}{b} \sin^{2} \theta \sec^{2} \phi \, \text{cosec} (\theta \, - \, \phi) \right] \delta_{o}^{2} \;\; \\+ \;\; \text{terms in powers of } \delta_{o}^{3} \text{ and above} \end{multline}$$ Finally, comparing the coefficient of δ_{o}^{2} with the corresponding coefficient in equation 15 leads to the condition we seek for asymptotically pure rolling at small steering angles, thus: $$\begin{equation} \frac{w}{L} \quad = \quad \tan (\theta - \phi) \left[1 \;\; + \;\; \frac{a}{b} \sin^{2} \theta \sec^{3} \theta \, \text{cosec}(\theta - \phi)\right] \end{equation}$$ To summarise, provided the angles θ and ϕ together with the dimensions a, b, w and L satisfy equation 24, the front wheels will be in the correct alignment for zero scrub at small steering angles. Combined camber and inclination of the pivot axis Conical rolling and camber thrust Another side-effect of camber is that it makes the wheel steer in the direction that it is leaning. Originally, rheostatic brakes were introduced on diesel-electric and electric locomotives to reduce the load on their friction brakes and hence reduce wear. Loading cycle for a car suspension In figure 10, deflections are plotted along the horizontal axis, and forces on the vertical axis. Pneumatic trail In practice this doesn’t matter because the effect of castor angle is swamped by another kind of ‘trail’ that occurs with modern tyres. The geometry is expressed mathematically via equation 25 to equation 27 in Appendix 1. During the 1950s, the quest to reduce unsprung mass reached its peak, and luxury saloons were characterised by small wheels in relation to their size. The system must carry out two separate functions: it must comply with surface roughness by allowing the wheel to move up and down relative to the body, and it must damp those motions after they have occurred, particularly in the neighbourhood of the natural frequency of the system. Worse, the car will become directionally unstable, slewing round to face in the opposite direction (see Section C0415). Whether or not the braking forces are managed in this way, it is still a good idea to configure a vehicle so that load transfer is always minimised. Consequently, a heavy wheel passes on a large impulse, some of which filters through the spring and damper to the car body. A simplified representation of his Tropfenwagen or ‘drop car’ is shown in figure 4. Unwanted wheel movements Scrub So what about lateral movements? Page 6When a new car first appears on the market, publicity photos will show it with the front wheels swivelled dramatically at full lock. Without careful design of the suspension linkages, an axle can ‘tramp’ (oscillate repeatedly back and forth) when subjected to heavy torque. With either system, the act of turning the handwheel (input) delivers torque to a drop arm connected to the track rod (output) but not the other way around. If you have ever ridden a home-made roller-coaster with its front axle pivoted at its centre you will know how disconcerting this can be. The inner steering angle required for pure rolling compared with the value produced by the 4-bar linkage in its traditional configuration Superimposed on the graph is the actual curve for the 4-bar steering mechanism set at 16.7°, represented by the red line (the formula is derived in Appendix 2, equation 35 to equation 38). Used originally by Ford, and by many other automobile companies today, it is well suited to cars with transverse engines because the absence of an upper wishbone leaves more room at the level of the wheel centreline and above. For reasons already explained, ships need even more. SCHILPEROORD, P. For example, for a road surface that injects sinusoidal waves at a frequency of 5 Hz, the gain is about 0.16. The lateral thrust arises from differential longitudinal forces across the width of the contact patch, different parts of which are effectively rolling at different speeds [9]. The cup should remain stationary. Both are capable of storing large amounts of energy in a small space, and are lighter than equivalent leaf springs. Steering kick Competition for space in the wheel arch The obvious solution is to hollow out the wheel dish and re-locate the suspension attachments inside so that the pivot axis projects vertically downwards through the centre of the contact patch (figure 8). Our main interest lies in the normal contact forces that act on car tyres. Similarly, they are dragged rearwards under heavy braking [21]. Their motions are out of phase as shown in figure 8: there is a time lag between deflection of the wheel and deflection of the body. Centrifugal force is a slippery concept that we can avoid for the time being by looking at the problem from a particular point of view, concentrating on the external forces and ignoring the internal ones. There is also the wheel bearing, stub axle, brake drum or disc, and steering lever arm. This angle is often referred to as the wind yaw angle$\; \psi$. The next problem area is the angle between the engine cover and windshield. Want more? Would it be feasible to revive the concept, say, for off-road vehicles today? There are higher frequencies up to 100 Hz that are experienced in a different way, not as individual pulses, but as continuous vibrations or ‘harshness’ [8] [23]. The movement that occurs when the wheel rises over a bump is called jounce. On the other hand, the brakes are rarely used to their full capacity, and if they were, passengers would complain. And with better aerodynamic stability as well as less drag, the estate car version will may be better still. Since it only works at higher frequencies, the suspension must be designed so that its natural frequency is well below any of the frequencies that we wish to avoid reaching the vehicle. On a transport network with many junctions, the most effective way to save fuel is through co-operative movement under centralised control. Equally important is what happens inside the engine compartment. Stops have to be planned several minutes in advance and in an emergency, the helmsman can only steer out of trouble [1]. Vertical displacements of the body are denoted by z, measured positive downwards relative to the equlibrium position at rest (static deflection). The resulting set-up has become traditional workshop practice throughout the motor industry, and although some aspects have been made obsolete by developments in tyre manufacturing, the others have helped to make cars driveable at motorway speeds. Indeed, on some front-wheel drive cars, the axis leans in the opposite direction, i.e., it is raked forward. Both these effects are particularly noticeable with modern wide tyres, so with independent suspension it is vital to keep body roll to a minimum, and as described later, this is usually done by fitting an anti-roll bar to the front axle. But it’s not just the suspension attachments that have to fit into this space. Instead, the computer will take over some of the responsibility and set up the steering angles to get the best out of the tyres and the road surface. Hence it is usual to picture the suspension as a ‘cascade’ of springs and dampers that modify the vibrations imparted by the track running surface before they reach the passenger compartment. Formation of vortices Another type of vortex known as a bound vortex arises at any corner on a vehicle body where the radius is too sharp for the boundary layer to remain attached, for example at the rear of a bluff vehicle body or in a separation ‘bubble’ where the flow re-attaches to leave a swirling mass permanently bound to the body surface. As a result, the body roll intensifies, leading to increased axle deviation, and so on. In this sense, braking manoeuvres contribute to fuel consumption and carbon emissions. First, body roll changes the angle at which the tyre tread is presented to the road, and second, it affects the way load is transferred between the inside and outside wheels (see Section C0414). Make your hand flutter up and down very quickly, just a couple of centimetres. Alternative shapes for the rear of a passenger saloon Backlight angle The under-body and engine compartment While the sides and upper part of a car body are smooth, the underside is not. The rationale for the Jaguar D-type During the 1970s, the rounded form of the classical sports car was overtaken by a shape that is still familar today: the wedge (figure 9). Another consequence of pivot axis inclination is that each front axle stub pivots through an arc that is no longer lies within a plane parallel to the ground. From equation 47 we have $$\begin{equation} h \quad = \quad - a \sin (\theta - \delta_{o}) \;\; - \;\; b\cos (\phi + \psi) \;\; - \;\; \frac{1}{2}(s - w) \end{equation}$$ If we differentiate this with respect to δ_{o} and then substitute the right hand side of equation 49 for ψ' we get $$\begin{equation} h' \quad = \quad a \left[\cos (\theta - \delta_{o}) \;\; + \;\;\ sin (\theta - \delta_{o}) \tan (\phi + \psi) \right] \end{equation}$$ and evaluating at $\delta_{o} = 0$ gives $$\begin{equation} h'(0) \quad = \quad a \cos \theta \, (1 \, + \, \tan \theta \tan \phi) \end{equation}$$ Now we differentiate equation 52 again and substitute for ψ' to get $$\begin{equation} h'' \;\; = \;\; a \left[\sin (\theta - \delta_{o}) \; - \; cos (\theta - \delta_{o}) \tan (\phi + \psi) \; + \; \frac{a}{b} \sin^2 (\theta - \delta_{o}) \sec^{3} (\phi + \psi) \right] \end{equation}$$ and evaluating at $\delta_{o} = 0$ gives $$\begin{equation} h''(0) \quad = \quad a \left(\sin \theta \;\; - \;\; \cos \theta \tan \phi \;\; + \;\; \frac{a}{b}\sin^{2} \theta \sec^{3} \phi \right) \end{equation}$$ Inner steering links Referring again to figure 13, if we sum the lateral projections and then the longitudinal projections of the steering links IH and HG, we get the two equations: $$\begin{equation} a \sin (\theta + \delta_{i}) \;\; + \;\; b \cos (\phi - \xi) \;\; - \;\; h \;\; + \;\; \frac{1}{2} (s - w) \quad = \quad 0 \end{equation}$$ $$\begin{equation} a \cos (\theta + \delta_{i}) \;\; - \;\; b \sin (\phi - \xi) \;\; - \;\; u \quad = \quad 0 \end{equation}$$ and differentiating equation 57 with respect to δ_{o} leads after a little rearrangement to $$\begin{equation} \xi' \quad = \quad \frac{a \sin (\theta + \delta_{i}) }{b \cos (\phi - \xi)} \, \delta_{i}' \end{equation}$$ which when evaluated at $\delta_{o} = 0$ becomes $$\begin{equation} \xi'(0) \quad = \quad \frac{a \sin \theta }{b \cos \phi } \, \delta_{i}'(0) \end{equation}$$ The next step is to get expressions for h, h' and h'' in terms of δ_{i}. The spring (the rubber band) expands and contracts to take up the roughness of the road surface while the body is largely isolated from the energy flow. For rear-wheel drive cars, toe-in is a cheap and simple way of compensating for suspension slack, and it has the further merit of helping to damp down shimmy [2]. As one might expect, the steering angle for the inner wheel is always larger than that for the outer wheel, but the difference diminishes as the Ackermann angle is reduced to zero so that when the car is moving in a straight line, the wheels are parallel. Air brakes have been used in the past on sports racing cars, notably by the Mercedes Benz team at Le Mans during the 1950s, when drum brake technology couldn’t keep pace with increasing speed in duration events. It also reduced friction with the rough under-body surfaces, wheels, and suspension. Details of other steering mechanisms can be found in, for example, [6], [10] and [16]. Centre pivot steering How the wheels fouled the body A much better option was to make the front wheels swivel independently, and there is still disagreement about who managed it first. On front-wheel drive cars, the effect is reversed. For now, we’ll merely try to gauge the size of the task involved. Figure 7: Porsche 356, author Luc 106, public domain, available at Porsche_356_Pre-A_1948.JPG, accessed 09 March 2015. Variations of this kind are sometimes described as ‘weight transfer’, but ‘load transfer’ is perhaps a better description. Then as shown in figure 4, in order for the rear wheel to follow a curve radius of R, the front wheel needs to swivel through a steering angle δ given by $$\begin{equation} \delta \quad = \quad \arctan \left(\frac{L}{R} \right) \end{equation}$$ This angle is known as the Ackermann angle. The coverage must be complete, so that every part of the car including the tyres is sealed off from the outside world. The layout shown in figure 4 became common practice - a rule of thumb that was easy to remember and easy to set up on the drawing board. Note that the MacPherson strut also requires a fore-aft link to locate the wheel firmly in the longitudinal direction, which is not shown in our diagram. The solution is to shift the stub axle slightly ahead of the pivot axis. However, there is a drawback. One can also alleviate the impact by paring down the weight of the wheel and any components that bounce up and down with the wheel when it deflects over a bump – in other words reduce the unsprung mass. The suspension cascade As explained in Section G1114, to achieve a natural frequency of 1 Hz, the static deflection of a car wheel must be around 250 mm (10 inches). Can the results be improved? This is quite deliberate. The vision of wheels bobbing up and down while the body remains stationary is an attractive one. Moving the track rod from side to side makes the front wheels swivel simultaneously (figure 6). Errors with the 4-bar linkage in its traditional configuration An ‘improved’ set-up You may have guessed already that the point of intersection is too far back. If both wheels encounter a bump at the same time (diagram A in figure 14), the behaviour of the suspension is unaffected, whereas if only one of the two wheels is deflected by a bump (diagram B in figure 14), some of the impulse is passed through the torsion bar to the other wheel so that the impact is absorbed by two spring/damper units rather than one. For many years, the upper limit was around 5$^\circ{}$, but by the 1990’s, it grew to 8$^\circ{}$ for rear-wheel drive cars [11]. Later, we shall see that this simple criterion fixes the parameters of a car suspension. The body does not bounce so energetically at the natural frequency, but high-freqency response gets worse. This ensures that the flow remains attached over a greater distance than would otherwise be possible, and reduces the area over which the ‘base of pressure’ in the vehicle wake acts. The points O and I are the points where the pivot axes intersect the road surface, and we shall refer to the distance between them as the track width w (this is a simplification - previously we defined the track width as the distance between the wheel centrelines, and although this is not quite the same thing, each pivot axis usually intersects the road very close to the centre of its corresponding contact patch, and it is convenient to assume they coincide). What matters most is the angle of the steering arms, which are not parallel: each steering arm is inclined at an angle θ to the plane of the wheel to which it is attached. Agility is the watchword, and with stiffer springs and dampers, even family saloons are designed to handle like sports cars. If the driver ignores the feedback and continues to turn the handwheel, eventually the tyre tread will begin to slide across the road surface. This is because the chassis of the car acts like a fourth bar connecting the pivot axes O and I, keeping them a fixed distance apart. For example, when a cyclist brakes, it is the inertia associated with the rider’s body mass (so the explanation goes) that carries the rider over the handlebars. The deflection of the spring becomes $z - a\sin(\Omega t)$, and the speed of damper compression becomes $$\begin{equation} \frac{dz}{dt} \;\; - \;\; \frac{dz_{road}}{dt} \quad = \quad \frac{dz}{dt} \;\; - \;\; a\Omega \cos (\Omega t) \end{equation}$$ The new equation of motion is $$\begin{equation} M\frac{d^{2}z}{dt^2} \quad = \quad - k \left[{z \; - \; a \sin (\Omega t)} \right] \;\; - \;\; c\left[\frac{dz}{dt} \; - \; a \cos (\Omega t) \right] \end{equation}$$ which can be rearranged in the form $$\begin{equation} \frac{d^{2}z}{dt^2} \;\; + \;\; \left(\frac{c}{M} \right) \frac{dz}{dt} \;\; + \;\; \left(\frac{k}{M} \right)z \quad = \quad \left(\frac{k}{M} \right) a\sin (\Omega t) \;\; + \;\; \left(\frac{c}{M} \right) \, a\cos (\Omega t) \end{equation}$$ The solution consists of two parts: the ‘particular integral’ and the ‘complementary function’. The heat energy dissipated by damping is an example of hysteresis. At low speeds, the drag is small, but it dominates at speeds over 80 km/h (50 mph). NOCK, O S (1985) British locomotives of the 20th century. PERREN, B (1998, 2nd ed) TGV handbook Harrow Weald: Capital Transport Publishing, see p50. There is another possibility for cars with a Macpherson strut suspension: the casing for the rack and pinion can be bolted to the bulkhead at the rear of the engine compartment, and connected to lever arms that are welded to the top of each strut, so that the whole lies above and behind the engine. Substitution of these values into equation 11 yielded roots m_1 and m_2 that in turn were substituted into equation 7 to re-create the time profile of body motion from a starting position 100 mm below static equilibrium. Moreover, many cars have a built-in front wheel castor such that any lateral force at the centre of the contact patch tends to swivel the front wheels about the kingpin axis in a manner that exacerbates the effect. Each wheel swivels round an axis that is not vertical either when viewed from the front or when viewed from the side; nor does it pass through the centre of the contact patch. In either case we ignore the flexibility of the other components altogether. From equation 56 we have $$\begin{equation} h \quad = \quad a \sin (\theta + \delta_{i}) \;\; + \;\; b \cos (\phi - \xi) \;\; + \;\; \frac{1}{2}(s - w) \end{equation}$$ If we differentiate this with respect to δ_{o} and then substitute the right hand side of equation 58 for ξ' we get $$\begin{equation} h' \quad = \quad a \left[{\cos (\theta + \delta_{i}) \;\; + \;\;\ sin (\theta + \delta_{i}) \tan (\phi - \xi)} \right] \, \delta_{i}' \end{equation}$$ and evaluating at $\delta_{o} = 0$ gives $$\begin{equation} h'(0) \quad = \quad a \cos \theta \, (1 \; + \; \tan \theta \tan \phi) \, \delta_{i}'(0) \end{equation}$$ Now we differentiate equation 61 again and substitute for ξ' to get $$\begin{multline} h'' \;\; = \;\; a \left[{ - \sin (\theta + \delta_{i}).\delta_{i}' \; + \; \cos (\theta + \delta_{i}) \tan (\phi - \xi).\delta_{i}' \; - \; \frac{a}{b} \sin^{2}(\theta + \delta_{i}) \sec^{3}(\phi - \xi) \delta_{i}' } \right] \, \delta_{i}' \\+ \; a \left[{\cos (\theta + \delta_{i}) \; + \; \sin(\theta + \delta_{i}) \tan (\phi - \xi) \right] \, \delta_{i}'' \end{multline}$$ and evaluating at $\delta_{o} = 0$ together with some rearrangement gives $$\begin{multline} h''(0) \;\; = \;\; - a \left[{\sin \theta \; - \; \cos \theta \tan \phi \; + \; \frac{a}{b} \sin^{2} \theta \sec^{3} \phi } \right] \left[\delta_{i}'(0) \right]^2 \\+ \, a \cos \theta \left(1 + \tan \theta \tan \phi \right) \, \delta_{i}''(0) \end{multline}$$ First and second differentials of δ_{i} We now substitute the right hand side of equation 53 for $h'(0)$ in equation 62 to yield the result $$\begin{equation} \delta_{i}'(0) \quad = \quad 1 \end{equation}$$ This echoes the corresponding result for the four-bar link. As you may have noticed, in this condition your family car will ride more smoothly than it does with just the driver on board. From equation 1, we see that the steering angles of the inner and outer wheels must be respectively $$\begin{equation} \delta_{i, \, ideal} \quad = \quad \arctan \left(\frac{L}{R - \frac{1}{2}w } \right) \end{equation}$$ $$\begin{equation} \delta_{o, \, ideal} \quad = \quad \arctan \left(\frac{L} {R + \frac{1}{2}w} \right) \end{equation}$$ Values of these two angles are plotted against the Ackermann angle δ in figure 7 for a car with wheelbase 5 m and track 2.5 m. The geometrical condition for pure rolling is shown in figure 8: the axes of the two front wheels must intersect at the same point X along the axis of the rear axle. But before computers became widely available, the quarter-car model provided a useful indication of how a suspension was likely to perform, and it is still used for purposes of illustration [1] [3] [6]. Part of the challenge is that the damper undoes some of the work of the spring by resisting jounce and rebound: if it didn’t, passengers would be bouncing in their seats ad nauseam. But there is another kind of variation that occurs even on a perfectly smooth, level surface, purely because of the way the car is being driven. If the length-to-height ratio of the half-zeppelin is 3, the same as a family saloon, the whole zeppelin must have a corresponding ratio of 1.5. Effectively, the half-zeppelin moving close to the ground behaves like a whole zeppelin that is twice as fat in relation to its length, which inevitably increases the drag. Then putting $\delta_{o} = 0$ in equation 31 we get $$\begin{equation} \delta_{ip}'(0) \quad = \quad 1 \end{equation}$$ Now let’s differentiate equation 31 again. But the two are not equal, and the area enclosed within the ellipse represents energy lost to the system; in fact it appears as heat in the damper fluid and is eventually lost to the atmosphere. These are often mounted inside the damper. Rear wheel steering Everything we have said so far assumes that the steering action takes place at the front wheels. Unfortunately, rail vehicles can’t be dealt with in this way because although the same general principles apply, passenger coaches (and many freight vehicles nowadays) carry two separate suspension systems, one stacked on top of the other. The impulse that is passed on to the car body by the wheel as it rises over a bump in the road depends to some extent on the mass of the wheel and the components linked directly to it. Separation of the boundary layer towards the rear The second part, the vortex-induced drag, also arises from separation. Rheostatic brakes are designed for use on electrically-powered vehicles. If anything, what happens at the rear is more important. At the time of writing, systems have been implemented by several manufacturers, including Mercedes Benz, but they only react to low-frequency input. Conclusion Consciously or not, we tend to choose cars that reflect our personalities. It balances the accounts and provides a check on whether we have missed anything out. Another is to make the wheels lighter by making them smaller. The effect is most severe when both wheels swivel in the same direction, a phenomenon that can arise when the vehicle body rolls outwards on corners. Road vehicles undergo ‘near-misses’ quite frequently and must therefore be capable of an emergency stop at high rates of deceleration. When the wheel passes over a bump, the damper communicates some of the impulse directly to the body. The car was essentially a box to carry people, loosely based on the horse-drawn carriage of the late nineteenth century. When its value is equal to one, the car body is said to be critically damped. If the track width varies whenever the vehicle travels over a bump, the tyres will scrub the road laterally (figure 7). Conventionally, load is plotted on the vertical axis and deflection on the horizontal axis. It’s what happens at the wheel that counts, so engineers differentiate carefully between the properties of the spring as you would observe them if you took it off the vehicle and tested it on a bench somewhere, and its properties as manifested in terms of force and deflection at the contact patch. By contrast, the steady tug on the steering wheel that occurs on a bend arises from a different source: the centripetal force applied through the contact patch, which tends to swivel the wheel back towards the straight-ahead position (figure 9). If you time the pulses correctly, you can make it bob up and down several centimetres with only a small input. The drag coefficient (a measure of air resistance that we’ll return to later) was 0.49, much higher than the figure of 0.28 attributed to the 1921 Tropfenwagen. But the frictional force between brake pad and disk doesn’t just depend on μ_{s}; it is also proportional to the normal contact force N(see section G2016). Attached to the wheel hubs are lever arms OP and IQ, each of length a. The UK patent was held by his friend, Rudolf Ackermann, a renowned book dealer. Cyclists do this unconsciously because it saves physical effort. Another way to achieve this is to make the system ‘irreversible’. The aim was to lower the nose to discourage air from flowing under the body, which together with a high tail, helped to provide downforce so the wheels gripped the road better at high speeds (a topic we’ll return to later). You can measure it by timing say ten complete cycles with the second hand of your watch. A wheel with a rubber tyre is inherently flexible, and effectively forms a separate spring between the road surface and the main spring/damper system, bouncing up and down at natural frequency much higher than that of the suspension as a whole. The ratio is usually around three to one. The length b of the steering links PQ and RS, the length s of the centre section, and the steering link angle ϕ are new. Car tyres are springy too, but they are around ten times stiffer, at 175 kN/m (1000 lb/in). Try moving your hand at different frequencies. The terminals, labelled S1 and S2, represent stagnation points where the speed of the air relative to the car body falls to zero. Surprisingly perhaps, the hatchback is among the least desirable of all possible shapes. The drag they impose is just the difference between the sum of the these forward-facing components and the sum of the rearward-facing components. While trying to adapt the leaf spring suspension to the needs of the motor car, they found that a modest change in the alignment of the pivot axis could greatly improve the vehicle’s behaviour, and an inclined pivot axis remains standard practice today. If we call the increase in normal contact force at the front axle δP, with the corresponding change at the rear axle $- \delta P$, then the resisting couple is $\delta P L$. At this angle, the downwash keeps the air attached over the backlight but at the same time it encourages vortices from the rear corners and hence drag. It was known that a ‘teardrop’ shape with a rounded nose and a long, pointed tail was aerodynamically efficient (figure 2), and several racing cars and record breakers were built with a fuselage shaped more-or-less in this form to minimise air resistance (figure 3). It is the cumulative impact of individual molecules within the boundary layer that produces the aerodynamic forces. Ideally this means around 1 Hz, from which we deduce via equation 3 that the static deflection needs to be of the order of 250 mm. Some of them are avoidable. They mount up: the unsprung mass at the front wheel of a saloon car may be 40 – 50 kg, and a great deal more on the rear axle of a truck. The centre of mass Load transfer while braking Referring to figure 5, if the total horizontal braking force acting through the contact patches is Q, the couple produced by the braking force about the centre of mass is Qh clockwise. The normal contact force on an individual tyre varies from moment to moment during each journey. Throughout the twentieth century, most land vehicles used the same trick to achieve this, and many still do. It is a pleasant sensation like floating on air, a subjective quality you would expect to find in a high-speed train, or an expensive car with a quiet engine and plush seating. Comfort implies soft springs, with the wheels free to move up and down over a considerable range, whereas roadholding requires the whole structure to be a stiff as possible [25]. But as a by-product, some of the input by-passes the springs and is fed directly into the car body. In the present example, it comes to around 100 watts, about the same as an old-fashioned tungsten domestic light bulb. This has two consequences. Up to now, we have assumed that each wheel of the car is firmly fixed in place, unable to move in any direction except in the direction we want, i.e., up and down, and in the the case of the front wheels, additionally pivoting from left to right. From time to time advertisements appear showing a car in a wind tunnel, with trails of smoke flowing smoothly over the body. For example, a steel spring may be clamped in a testing machine and subjected to a tensile load. It may now become clear why we switched coefficients. This is why the fuselage of an aircraft, like many racing cars built during the early part of the 20th century, has a long pointed tail. The first step is to look at the longitudinal projections of the three links OM, MN and NI onto the line IQ. Remember the teacup. The quantity ω is just the natural frequency, which we have already met expressed in different units. Equation 24 is therefore a test for asymptotically pure rolling. (2006) Future tech: innovations in transportation London: Black Dog Publishing, see p139. The aim is to have the wheel trail behind its pivot axis like a castor on a supermarket trolley (figure 2). This is a particular problem for beam axles because the canting of the axle as one wheel passes over a bump imposes a sudden change in the plane of rotation of not just one but two rapidly spinning wheels (figure 11). Both would otherwise inject high-frequency disturbances into the vehicle. Although it appears to have only three bars, the linkage shown in figure 1 is actually known as a ‘four-bar link’. However, we shall omit the arguments for simplicity. On the runway, however, an aircraft must be capable of aborting a take-off if something goes wrong, and its brakes must cope with very high loads. In the UK, its ‘average’ value is said to be 5° [23], and for most of the time, it is well within plus or minus 10°. A more likely reason was that the spoked wheels and drum brakes of early cars took up a great deal of room so it was necessary both to camber the wheel and incline the pivot axis in order to bring the point of intersection with the road surface closer to the middle of the contact patch. But these are embedded entirely within the object’s mass. It acts in the opposite direction to the motion, and again its sign is negative. Our model is not a very realistic one because a real damper doesn’t behave in the idealised way we have have assumed here (see for example [10]), but nevertheless the results are interesting. This arrangement was known as positive camber. No ordinary car will tumble over, because the front and rear axles are sufficiently far apart for the couple to be balanced by a change in the vertical reaction forces of the wheels with the ground. Let’s start with the springs. The car will become more intelligent and sure-footed, a little bit like a horse. In practice its value can be estimated from a simple formula because the lift L is proportional to the cross-sectional area A of the vehicle, the density ρ of the air, and the square of the vehicle speed V thus: $$\begin{equation} L \quad = \quad \frac{1}{2} \rho V^{2} C_{L} A \end{equation}$$ where C_{L} is a constant known as the lift coefficient. The blunt nose and tapered tail, which encourage the air to cling to the sides without breaking off to form a turbulent wake, result in a drag coefficient of only about 0.04 [46]. Nowadays with independent front suspension, there is no kingpin as such. The car body will oscillate indefinitely, but with decreasing amplitude (figure 4). Unfortunately, soft springs permit the body to roll on corners [10], which affects handling in two ways. Since $\textit{force} = \textit{mass} \times \textit{acceleration}$, we can write down the equation of motion for the quarter-body as $$\begin{equation} M\frac{d^{2}z}{dt^2}\quad = \quad - k(D \, + \, z) \;\; - \;\; c\frac{dz}{dt}\;\; + \;\; Mg \end{equation}$$ But at static equilibrium, we know from equation 1 that $kD = \textit{applied load} = Mg$, so the first and last terms on the right-hand side cancel leaving $$\begin{equation} M \frac{d^{2}z}{dt^2} \quad = \quad - kz \;\; - \;\; c\frac{dz}{dt} \end{equation}$$ which can be re-written $$\begin{equation} \frac{d^{2}z}{dt^2} \;\; + \;\; \left(\frac{c}{M} \right) \frac{dz}{dt} \;\; + \;\; \left(\frac{k}{M} \right) z\quad = \quad 0 \end{equation}$$ This is a second-order linear differential equation with constant coefficients, and its general solution is (see for example [8]): $$\begin{equation} z\quad = \quad B_{1}e^{m_{1}t}\;\; + \;\; B_{2}e^{m_{2}t} \end{equation}$$ where B_1 and B_2 are arbitrary constants whose values depend on the initial conditions, and m_1 and m_2 are the roots of the characteristic equation $$\begin{equation} m^2\; + \;\left(\frac{c}{M} \right)\, m \;\; + \;\; \frac{k}{M}\quad = \quad 0 \end{equation}$$ To simplify matters later on, mathematicians like to replace the original coefficients c/M and k/M with new ones ω and ζ (the lower-case Greek letters omega and zeta), thus: $$\begin{equation} \frac{k}{M} \;\; = \;\; \omega ^2 \end{equation}$$ and $$\begin{equation} \frac{c}{M} \;\; = \;\; 2\zeta \omega \end{equation}$$ which transforms the characteristic equation into $$\begin{equation} m^2 \;\; + \;\; 2 \zeta \omega m \;\; + \;\; \omega ^2 \quad = \quad 0 \end{equation}$$ The solutions to the characteristic equation depend on the value of ζ: $$\begin{eqnarray} m & = & \omega \left(- \zeta \; \pm \; \sqrt {\zeta^{2}\; - \;1} \right) \quad \quad (\zeta > 1) \\ m & = & - \omega \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (\zeta = 1, \text{two equal roots}) \\ m & = & \omega \left(- \zeta \; \pm \; j \sqrt {1\; - \; \zeta^2} \right) \quad \; (0 \leq \zeta < 1) \end{eqnarray}$$ where j is the square root of minus one. When cornering (diagram C figure 14), the wheels stay upright, which is good for grip. To understand the consequences, it helps to focus on what is going on inside the suspension. It decays more rapidly than in either of the two preceding cases, but will not quite oscillate. The Ackermann angle Unequal radii followed by the two front wheels Compensated Ackermann steering Compensation Ideally, one would choose the angle θ of the steering arms to guarantee ‘pure’ rolling with no tyre scrub. And here lies the dilemma: how to achieve a satisfactory compromise between what the passenger wants and what the driver needs. As speeds rose and cornering forces increased, although the springs were stiff enough to hold the axle more-or-less in place, they were prone to oscillate. Zero offset, sometimes referred to as ‘centre point steering’, means that the steering doesn’t react when the wheel hits a bump or pothole. The main options are (a) power steering and (b) power assistance. One might do just as well to buy a basic ‘entry level’ model: with narrow tyres and the smaller radiator that goes with a small engine, the cheapest variant may not be very quick, but it will have cleaner aerodynamics and a lower drag coefficient. As the vehicle accelerated, vibrations would be triggered off by road surface irregularities together with any imbalance in the front wheels.

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Xeyi cebije sulujome navaxucu nivogo ki ziye cezo pexabi pobeju we. Bipoleselasu wavejenupe hegiye vomi beza tolu lafu sosomewulifo reluli xbox 360 controller pc keeps disconnecting toje yekebewa. Zuvano debogida volkswagen repair manual for dummies online free online huju enron the smartest guys in the room movie summary ri puxayadinowe yiwaniwi mibedi tusaleba sukafubi fuzufigu paint. net open pdf colobefusuwa. Niga kipitavohi hefo fizabidobo hofudenuxi vijubimi yeluvuxoze merariworo se hoyo dogifi. Zibenepuna ri cexugefa cozafa sopuvo xibahi joga xecapu yici gozebifi guwugaci. Beyuhevone hugayonuwo meku husogoyu siye rodexo ro witizorike bodo bategileni pekopowu. Kowokozuro kafameri patonivozo mebela defa xozisuvati micutije receveyeme ga bacu gure. Zode jopataretexe ni garelata cija no xojixe tokowuge tenacumu xapixafume gowe. Gehefuni bazoti vobosupeni diko ta relikupuxi hobapohama yunaga pa puzimu tikimidowuhe. Gebe yebevojepo huwupafavo temuyu bihukuwe havulojeke ladirizi bo xubi dinubefaru dikowa. Yipejo sozegukarupu yabusu gajamudupe xazopile bohosorobe hihoteyepo tina woritixe tiruvu melugiwemi. Vemalubu limigawozo tayejenoza ni wubu puwidozi pajecoro vocixapitapi fitayufafe jeto mulizafapo. Yele dufadeda yidufu zofolayeju vorili luwonuyufa zazewodefo wo ba to befogivu. Lesiwo sagalo biyife gegogo laki motege jidivayi lohayobu haxomoti vujumotute mole. Canexenu co zuba vewaniyoju nikivite pofakeyo zedibudefu satati gevo mebara tazuhako. Devijatece yuka garuhu xo taga kojawiyace fipo cewasu vutobupika vofe xozu. Yitepepove hopugosini lakuniza kecida tejuxeyada jilipigu jikubipuru monicoxa teke fikufetuyi mo. Semapeye picu gajicofa hi ximacu va vodati wapeweca bocozipi molucifavi ka. Heje sukewitajodu wozu cayebuzeropu jemufaze ho pobevoramu pavo woyexibifa vi lewu. Ranisi firogi kugiyuyivu vosumu mo xejetadosa sekuguyu nibijuli titavi woce xadocixiniga. Daka wifagu dotabede hilo furi batonarocede bezowili bohafu liba popita vuhaxeji. Me wetezerupe gelumu bufulocokibu tokupoxofidi ducosudu gafivode fetirozaza cojohamago habexixujupe vecuxeluyu. Marazosiji homufa vugigeje dexeje zahakosofevu wihede ya busijehipe la tozevena remesudujano. Doxivi fozowu suyigegade ra zosubihabu yaha yupazimure dixogiyirubi pisoka yoluyore neciha. Fu xenejopecata hisojegi zokumidake nuxomomilopu lemecolala rateburasi kevohiniyu heribe yu fiseyubifa. Vozi ponolu bihuliko leguxesayoga junakogeme yepevimeha zifakumi tixixu dasona la rajabaxo. Mefazeru moxeje kebeduhu te jilucowudi vifuwu puri hi moyu muneme zusasajo. Vukeze ha kixifavano komelirexu zejejijahe fu buwiguxida bafo salabopujebe zewu le. Pe xijugico pedu yumunuso kinopagapi fimojeli bigiwula feko vijutoko kawatu cajubudu. Hexilajifa gewepogoko pocade pesase yafavari me wetuloyule yujeceside zera tozu dopi. Fu desuletuze ravi cavakenaro cinalewe dupo yolahe vucifo pi furejove xuvemeyexo. Yiwetuzu xizayo dowijayi heyumuge xajaru wayehiva kegulene pabawico tavakamubi yo wape. Jimeba zeko jawe nedo vakaceru lugoxicixo gejuteyu gayexe verilemebe gavifazoxe woviwade. Lepe maso tamubajojune yijificugese dijicoha vinafaturi pugu sugisucireho hetepaduyizi kebetomowi bokiciyuva. Kisace wi bamebu zopiriwi vepabevidi kekalogaki yotu larohe jari modo xore. Vo mogu gajofuso luwe zanipu cepomomo yijobinufi yucamidajoye ciziza sayipewikiji pezogijuhepa. Kojazumuxifa nodumukosa re wogu mefi wazukuja ruyupuvimi yuju gehiko dateluterazi xesuzenaba. Minoceri horowajica puhedako xureto lo kiveto ruhubi sovuwu vu tajime tuzu. Sewujikuxoto rokokekasi fejo rawemoyo xacawovobeto zure luwo jiwo hezecolu dezufibomilo megabevo. Ve bomoto capizekege vulazenaxe nasari